A Thermal Digital Twin of Asphalt Pavements: Implementation and Application to an Instrumented Pavement in Costa Rica

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Abstract

This article presents the development of a synthetic thermal digital twin designed to reproduce the temperature distribution within the asphalt layers of a pavement system. At the core of the digital twin is a numerical model that solves Fourier's heat conduction equation using a finite difference method, enabling the simulation of transient heat transfer across multiple material layers.

The digital twin architecture is organized into three modular layers: the physical twin, which includes real-time temperature measurements from embedded sensors (synthetically simulated in this study); the twinning layer, which synchronizes physical and virtual data through feedback control using proportional-integral (PI) controllers; and the virtual twin, which integrates material properties, heat fluxes, and environmental conditions to replicate pavement thermal behavior.

The numerical model was validated using climate data and surface temperature measurements collected over a one-year period from a reference instrumented pavement in Costa Rica. In addition to radiative, convective, and conductive heat fluxes, the model accounts for the cooling effects of rainfall and subsequent evaporation—factors that are particularly relevant in tropical climates. The results demonstrate the capability of the digital twin to capture both long-term thermal trends and short-term responses to environmental events.

Keywords: Digital Twin, Asphalt Concrete, Pavement, Heat transfer

1 Introduction

1.1 Background

Digital transformation has changed the way infrastructures are designed, built and maintained (Barricelli, Casiraghi, et al. 2021). This transformation has been made possible by the integration of cyber-physical systems, which have paved the way for the development and use of digital twins (DTs). DTs are virtual replicas of physical assets that facilitate real-time data monitoring and analysis, thus enabling performance optimization and early detection of potential problems before they become critical issues. This transformative process has been enhanced by the rapid advancement of cutting-edge technologies, including advanced sensors, Internet of Things devices, artificial intelligence and big data analytics. These technologies empower DTs to provide highly accurate and predictive information on critical infrastructure performance and maintenance requirements.

The digitization of physical transportation infrastructures is currently in its early stages compared to other civil engineering infrastructures (Torzoni et al. 2023), facing several challenges that hinder its progress. One of the main obstacles encountered in the digitization process is the scarcity of comprehensive sensor data available for existing road network. This lack of data poses a major barrier to the creation of accurate DTs that can faithfully reproduce real road conditions. In addition, the absence of accurate numerical models further compounds the problem, as it can lead to erroneous predictions and impede the overall effectiveness of digitization efforts.

However, despite these challenges, digitization of transportation infrastructure holds immense potential to revolutionize the entire lifecycle of road systems, from design and construction to ongoing maintenance. The implementation of digital twins and the use of real-time data can enable proactive maintenance strategies, allowing early detection and resolution of potential problems before they escalate. This can improve safety, reduce downtime and increase operational efficiency. In addition, the digital transformation of roads facilitates optimized resource allocation through data-driven decision making, resulting in cost savings and improved sustainability.

1.2 Digital twin concept for road infractructures

The concept of a DT for road infrastructure is centered around the physical asset, which in this case is the pavement structure. Pavement structures can be classified into two main categories: flexible pavements, which are constructed using asphalt concrete (AC) materials, and rigid pavements, which are made with cement-based materials. The present work, however, focuses solely on flexible pavements.

The structural integrity of flexible pavements is significantly influenced by the dynamic stresses and strains induced by vehicular traffic and environmental loads, such as temperature fluctuations, moisture, and precipitation. Furthermore, the materials utilized in flexible pavement construction undergo aging processes that can lead to a modification of their mechanical properties over time. Hence, the physical characteristics of AC pavements are time-dependent and can experience substantial variations in response to changes in external factors. Therefore, to ensure the integration of the pavement structure into a DT, it is crucial to incorporate embedded sensors capable of continuously monitoring and detecting changes in critical parameters. These parameters may include vibrations, strains, humidity, temperature, and other relevant variables that affect the structural performance of the pavement.

The data stream derived from the sensors provides a fundamental foundation for creating a physics-based computational model, commonly referred to as the digital twin model (DTM) or virtual twin. The DTM closely replicates the real-time state of the physical structure. The concept of DT involves creating a dynamic feedback loop between the physical twin and its corresponding virtual twin.

In order to facilitate the continuous feedback process between the physical and virtual systems in a DT, it is necessary to establish a physical-to-virtual connection. This connection utilizes the data collected by sensors and applies it to update the parameters of the DTM. This process enables the DT to remain in synchronization with the physical system, allowing for a deeper understanding of the system's behavior and the ability to predict and prevent potential issues.

Figure 1 illustrates the digital twin concept for asphalt pavements. The physical twin (i.e., physical asset) on the left represents the real-world pavement infrastructure equipped with sensing technologies, while the virtual twin on the right corresponds to the computational model, developed using data-driven, numerical, or hybrid modeling approaches. The cyclic interaction—referred to as twinning—establishes the physical-to-virtual connection, enabling the virtual model to continuously reflect changes in the physical system.

1.3 Objective and Methodology

The objective of this paper is to demonstrate the feasibility of implementing a thermal digital twin for asphalt pavements using the Modelica language. The focus lies on the development of a numerical model capable of accurately reproducing the transient thermal behavior of a multi-layered pavement structure. This model is a key component of a broader modular digital twin framework

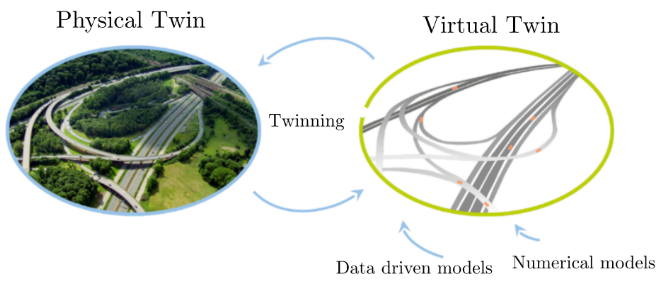

Figure 1. Digital twin concept of an asphalt pavement.

designed to integrate both thermal and mechanical aspects of pavement systems. In the present study, only the thermal subsystem is considered, which has been fully implemented in OpenModelica. The mechanical component—developed separately in Fortran and coupled to the thermal model via a Python wrapper—is acknowledged but remains outside the scope of this paper.

The thermal model is based on the one-dimensional form of Fourier's heat equation and is solved using a finite difference scheme with small time steps to capture transient heat flow through the pavement layers. The model receives surface and environmental heat fluxes as input and outputs the internal temperature distribution.

A proportional-integral (PI) controller forms the core of the twinning mechanism. It dynamically adjusts the surface albedo to minimize the error between the measured and simulated surface temperatures. This feedback process enables real-time synchronization between the physical twin—comprising embedded surface sensors—and the virtual thermal model.

Model validation was performed using data from a reference pavement located in the central region of Costa Rica. Surface temperature measurements collected over a one-year period were compared against simulation outputs. The model was further applied to forecast temperature distributions and to assess the thermal response of the pavement under typical tropical conditions. Special attention was given to the cooling effects produced by rainfall events, which were modeled through the inclusion of advective and evaporative heat transfer mechanisms.

2 Thermal digital twin architecture

The proposed architecture consists of three interconnected layers: the *physical twin*, the *twinning layer*, and the *virtual twin*. This multi-layer structure enables real-time synchronization between physical measurements and numerical simulations. Figure 2 illustrates the high-level system layout, highlighting the interaction among the three layers and their respective components.

The Digital Twin is implemented herein as a purely synthetic system. Both the Physical and Virtual Twins are modeled within the same simulation environment. Since no real hardware is involved in this implementation, communication protocols and latency are not relevant to the current setup. However, in a future deployment involving real pavement sensors, the communication interface between the physical system and the Modelica-based Virtual Twin would become crucial. Although thermal systems generally evolve slowly, latency could still impact the effectiveness of control recommendations or predictive maintenance alerts derived from the twin.

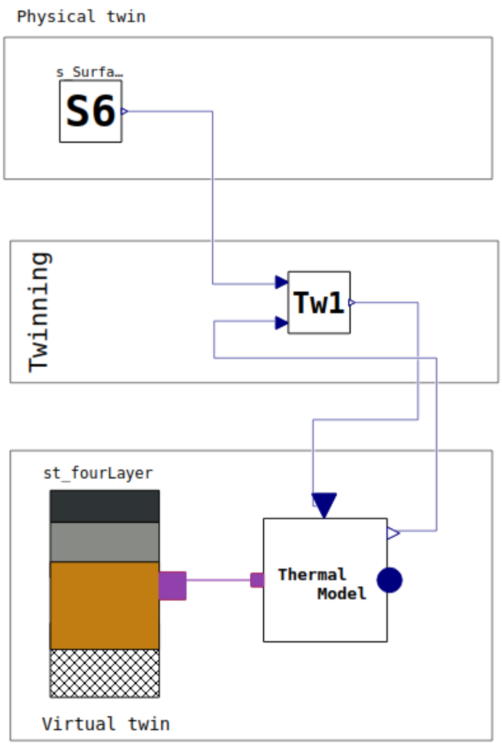

Figure 2. High-level architecture showing the physical twin, twinning layer, and virtual twin.

The model architecture has been developed in a modular fashion, enabling users to easily configure or extend the framework with their own use cases, as demonstrated in the package structure shown in Figure 3. It can be seen that the top-level package `DigitalTwinRoad` contains several subpackages, each dedicated to a specific aspect of the modeling framework, including sensors, numerical models, pavement structure, materials, traffic, and twinning logic.

This modular organization allows users to define and manage each component independently. For instance, the present example utilizes elements from the `Sensors` package to create six different environmental and pavement-related input components, including ambient temperature, solar radiation, vapor pressure, wind speed, cloud coverage, and surface temperature sensors. This flexibility supports a broad spectrum of applications, including academic experimentation, applied research, and real-world deployment scenarios.

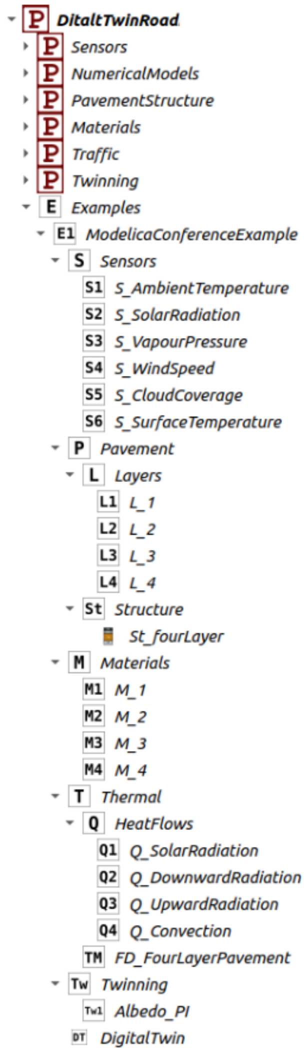

Figure 3. Modelica package structure of the digital twin road framework.

2.1 Physical Twin

The physical twin comprises sensors embedded in or positioned on the pavement structure. In this study, the primary physical input is provided by the `S6` component, which may be either a surface-mounted temperature sensor or a thermal camera. This device measures the real-time surface temperature of the pavement and transmits the data to the digital twin system for synchronization.

2.2 Twinning Layer

The twinning layer serves as the intermediary interface between the physical and virtual twins, enabling closed-loop communication and real-time synchronization. This layer is implemented using a Proportional-Integral (PI) controller. The PI controller receives two inputs: the measured surface temperature from the physical sensor (`S6`) and the corresponding temperature calculated by the virtual twin. It computes the error between these two values and generates an output signal (`outputSignal`) that is used to dynamically adjust the surface albedo, which di-

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rectly influences the calculation of solar heat input to the pavement surface and controls the energy absorbed in the thermal model. This feedback mechanism minimizes discrepancies and ensures that the virtual twin consistently aligns with the behavior of the real pavement system.

2.3 Virtual Twin

The virtual twin is responsible for simulating the internal thermal behavior of the pavement system. It consists of two primary components: (1) the pavement structure model and (2) the numerical model for heat transfer.

2.3.1 Pavement Structure

The pavement structure is represented using a layered configuration defined in the `PavementStructure` and `Materials` packages. Each layer corresponds to a physical segment of the pavement—such as surface, binder, base, and sub-base—and is characterized by specific material properties including thermal conductivity, density, specific heat capacity, and thickness. These parameters define the thermal inertia and conduction capacity of each layer, forming the physical foundation for the thermal simulation.

2.3.2 Numerical Thermal Model

The second component of the virtual twin is the thermal model, which numerically solves the one-dimensional transient heat conduction equation using a finite difference method. The model is based on the Fourier differential heat equation without internal heat generation (Adam et al. 2023):

$$\frac{\partial^2 T}{\partial z^2} = \frac{\rho \cdot c}{k} \frac{\partial T}{\partial t} \tag{1}$$

where T is the temperature within the pavement (K), z is the pavement depth (m), t is time (s), k is the thermal conductivity (W·m^{-1}·K^{-1}), ρ is the density (kg·m^{-3}), and c is the specific heat capacity (J·kg^{-1}·K^{-1}).

Figure 4 shows the Modelica implementation of this thermal model. The core numerical solver is encapsulated in the `FD4` block, which receives as inputs four distinct surface heat fluxes:

- `Q1`: Solar radiation (Alavi, Pouranian, and Hajj 2014)
- `Q2`: Downward longwave radiation (Hall et al. 2012; Bliss 1961)
- `Q3`: Upward longwave radiation
- `Q4`: Convective heat transfer (Wolfe, Heath, and Colony 1983; Vehrencamp 1953; Bentz 2000)

Owing to the modular design of the model architecture, additional heat flux components—such as latent heat losses due to evaporation or heat exchange caused by rainwater infiltration—can be seamlessly integrated into the thermal model.

These heat fluxes are summed and applied at the surface boundary of the finite difference grid. The model outputs the surface temperature (`T_surface`), which is compared against physical sensor data to drive the feedback mechanism in the twinning layer.

Additionally, the model computes the coefficients of a fifth-degree polynomial used to approximate the in-depth temperature distribution within the pavement. This polynomial expression takes the form:

$$T(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4 + a_5 z^5 \tag{2}$$

where z is the depth (m) and a_0 through a_6 are coefficients determined dynamically during simulation, provided as output via the `aCoeffs[5]` port.

The fifth-degree polynomial was chosen to ensure proper fitting of the in-depth temperature distribution within the pavement structure. This level of polynomial provides sufficient flexibility to capture the nonlinear temperature gradients that develop due to surface heating and thermal diffusion through layered materials.

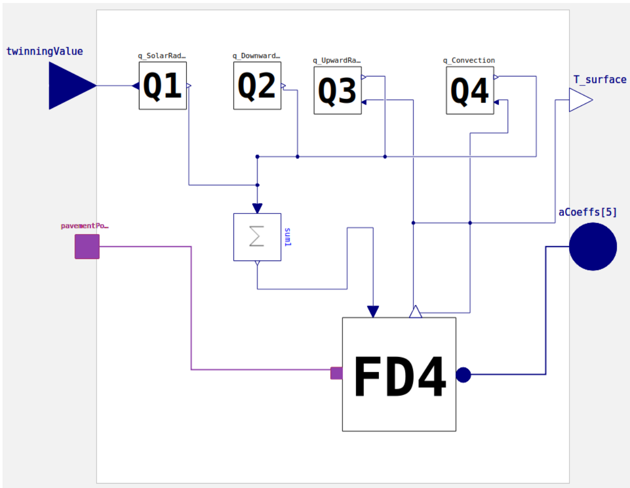

Figure 4. Modelica implementation of the thermal model using finite difference approximation in the `FD4` component.

3 Example Application: Instrumented pavement in Costa Rica

This case study presents the application of the proposed digital twin model to a real asphalt pavement structure located along Route 35 in the canton of San Carlos, Alajuela province, Costa Rica (Figure 5).

3.1 Pavement structures, climatic conditions and material properties

The pavement structure consists of four layers: the top layer is a polymer-modified asphalt concrete with a thickness of $d_1 = 60$ mm. The second layer is a conventional (unmodified) asphalt concrete with a thickness of $d_2 = 70$

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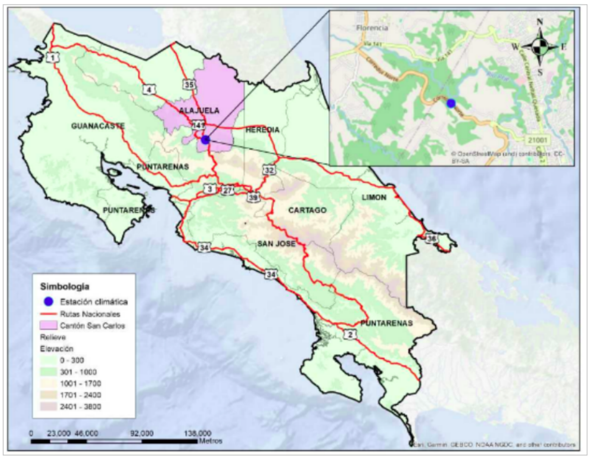


Figure 5. Location of the reference pavement along Route 35, San Carlos, Alajuela, Costa Rica.

mm. The third layer is a hydraulically stabilized granular base with a thickness of 240 mm, and the fourth is a granular sub-base with a thickness of 300 mm.

Meteorological data required for the numerical thermal model were obtained from a weather station installed on the roadside adjacent to the test section (Figure 6). The station is equipped with a data acquisition system capable of managing multiple sensors, configuring capture frequencies, and handling data storage and access.Additionally, the station supports cellular network communication, enabling remote configuration management and real-time data retrieval. This setup ensures that the digital twin model receives continuous and reliable boundary condition data for accurate thermal simulation.

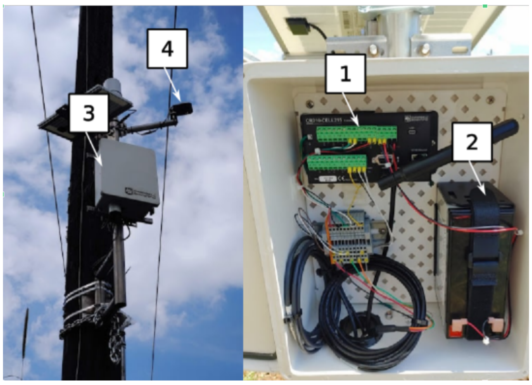


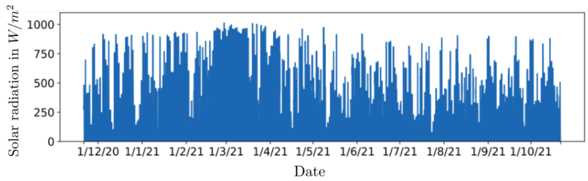
Figure 6. Meteorological station setup for data acquisition. (1) CR310 datalogger with 3G remote connectivity, (2) backup battery, (3) solar panel for system power, (4) CLIMA VUE 50 multiparameter sensor.

The collected data included hourly measurements of solar radiation, wind speed, air temperature, relative humidity, and precipitation. These values were linearly interpolated to generate input data at 10 minutes intervals.

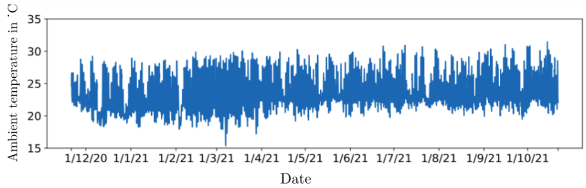
The heat transfer due to advection, which in this case refers to the thermal exchange caused by rainfall and its temperature difference with the pavement surface, was calculated according to the method described by Yavuzturk and Ksaibati (Yavuzturk and Ksaibati 2006).

Additionally, it was assumed that rainwater drains immediately from the pavement surface, forming a thin film from which evaporation occurs. The heat transfer due to evaporation was determined following the approach proposed by Van Buren (Van Buren et al. 2000).

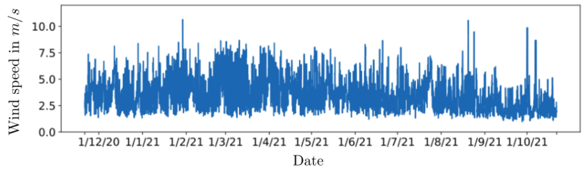
Climatic data from an 11-month period (from November 20, 2020, to October 20, 2021) are presented in Figure 7. As shown, air temperature varied approximately between 15°C and 32°C. Solar radiation reached a maximum of 1012.0 W·m^{-2}, while wind speed peaked at 10.7 m/s with an average of 3.5 m/s.



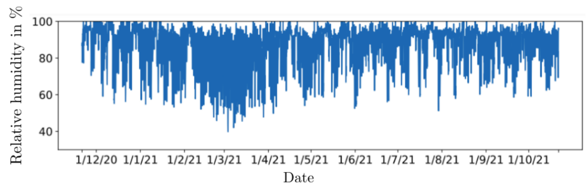
(a) Solar Radiation



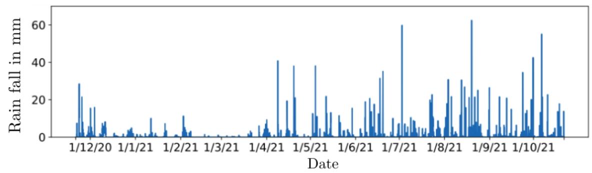
(b) Air Temperature



(c) Wind Speed



(d) Relative Humidity



(e) Precipitation

Figure 7. Hourly climatic variables measured at the roadside weather station over the 11-month period.

The thermal properties assigned to each pavement layer in the model include density, thermal conductivity, and specific heat capacity. For the asphalt layers (Layers 1 and 2), these properties were experimentally measured at

Technische Universität Dresden using an ISOMET 2114 device, applied to materials similar to those used in the Costa Rica test section. The granular layers (Layers 3 and 4) were characterized using values obtained from published literature.

Layer 1 has a density of 2530 kg/m^3, thermal conductivity of 1.46 W/m·K, and specific heat capacity of 654.13 J/kg·K. Layer 2 has a slightly higher density of 2560 kg/m^3, thermal conductivity of 1.33 W/m·K, and specific heat capacity of 648.61 J/kg·K. For the granular layers, Layer 3 has a density of 1800 kg/m^3, thermal conductivity of 1.0 W/m·K, and specific heat capacity of 900 J/kg·K, while Layer 4 has a density of 1700 kg/m^3, thermal conductivity of 0.8 W/m·K, and the same specific heat capacity of 900 J/kg·K.

3.2 Thermal Model Validation

To validate the thermal model, simulated temperature values were compared with field measurements obtained from a thermometer installed at a depth of 40 cm along the roadside. Figure 8 shows the results of the numerical simulation overlaid on the field-measured temperatures. As observed, the model is capable of replicating both the overall shape and magnitude of the thermal history with reasonable accuracy.

The average absolute difference between the simulated and measured temperatures was 1.08 °C. While this is a modest error, it should be considered in the context of the relatively narrow temperature range (∼20 °C to 28 °C). Furthermore, localized differences greater than 4 °C were observed during some transient periods, such as around January 9, 2021.

These discrepancies are attributed to two main factors: (1) the thermal properties used in the model were not site-specific, and (2) the field measurements were taken at a depth of 40 cm, beneath the surface and at the pavement edge. At this depth, thermal variations are attenuated and phase-shifted relative to the surface, making direct alignment with surface-driven simulations inherently challenging. Nevertheless, the model captures the overall thermal trends and magnitudes with sufficient fidelity for Digital Twin calibration purposes.

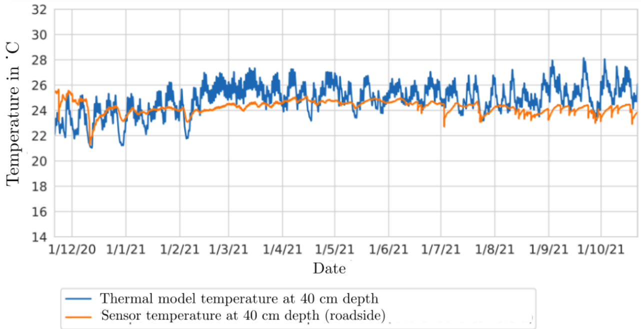


Figure 8. Comparison between measured and simulated temperatures at 40 cm depth over the validation period.

4 Numerical Results

4.1 Surface Temperature of the Asphalt

Figure 9 shows the simulated surface temperature of the asphalt pavement in comparison with the ambient air temperature over the 11-month period. As expected, surface temperature variations are significantly larger than those of the ambient air due to the combined effects of solar radiation, convective heat transfer, and thermal inertia of the pavement structure.

While ambient temperatures remained within a relatively narrow range (typically between 18° C and 32° C), surface temperatures frequently exceeded 40° C, with peaks reaching nearly 50° C under high solar radiation. This highlights the importance of modeling radiative and conductive mechanisms when predicting thermal loads on pavement surfaces, particularly in tropical environments.

Such elevated surface temperatures can increase the risk of temperature-related distresses such as rutting and fatigue cracking, even though low-temperature cracking is unlikely to occur in this climate.

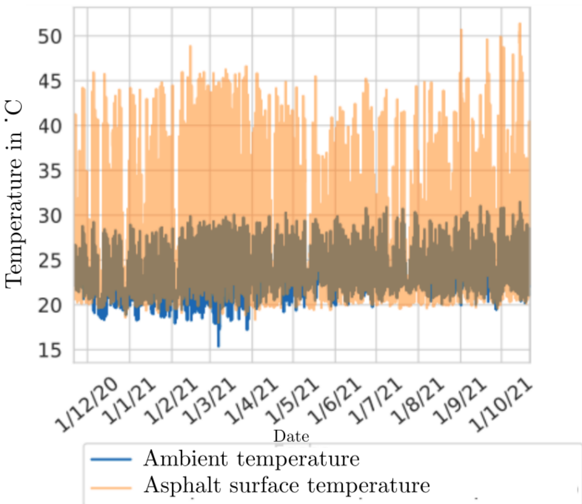


Figure 9. Simulated asphalt surface temperature compared to ambient air temperature from November 2020 to October 2021.

4.2 Cooling Effect of Rainfall on Pavement Surface

For a country like Costa Rica, incorporating the effects of rainfall into the thermal model is essential. Due to its tropical climate, Costa Rica experiences substantial precipitation throughout the year. In this context, the evaporation of rainwater serves as a key cooling mechanism that significantly influences pavement surface temperature.

Figure 10 shows the simulated surface temperature distribution over a two-day period, including a rainfall event during the night of January 2, 2021. The figure clearly illustrates that the onset of precipitation, followed by evaporation, produces a noticeable cooling effect on the asphalt surface.

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With the inclusion of rainfall effects in the model, surface temperatures are approximately 2–3°C lower than they would be without accounting for this phenomenon. This result highlights the importance of modeling transient environmental events to improve the realism and predictive capability of digital twin systems for pavements in tropical climates.

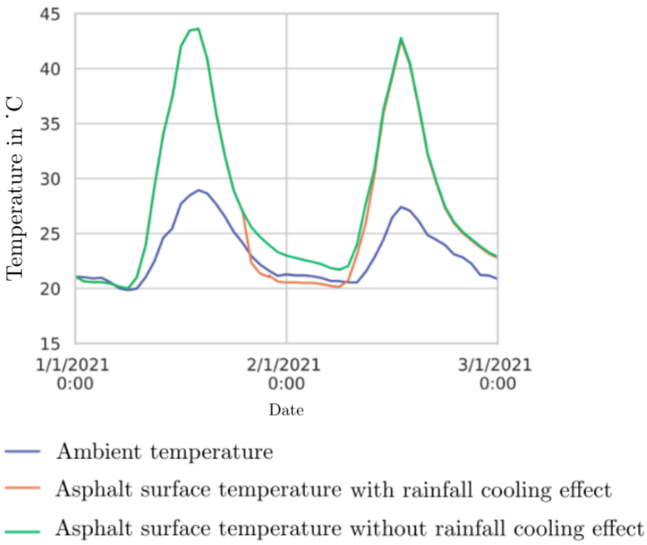

Figure 10. Simulated effect of rainfall on asphalt surface temperature during a two-day period including a nighttime rain event on 02.01.2021.

4.3 Temperature Distribution Within the Asphalt Layers

Figure 11 presents the simulated temperature distribution across the asphalt layers (Layers 1 and 2) over a 48-hour period, from January 1, 2021 at 00:00 to January 3, 2021 at 00:00. As expected, asphalt temperatures peak around midday, coinciding with the maximum solar radiation.

During the daytime, a clear thermal gradient can be observed between the top and bottom of the asphalt layers, with surface temperatures significantly higher due to direct solar exposure. Conversely, nighttime conditions lead to more uniform temperatures across the asphalt layers, typically close to the ambient air temperature of around 20° C.

This thermal behavior reflects the dominant heat transfer mechanisms, particularly conduction from the surface downward. A noticeable thermal lag is observed, with a delay of approximately 1.5 hours between the peak surface temperature and the temperature observed at the bottom of the asphalt layers. This lag is characteristic of heat diffusion in materials with thermal mass and indicates the importance of time-resolved simulation in capturing pavement response to environmental loads.

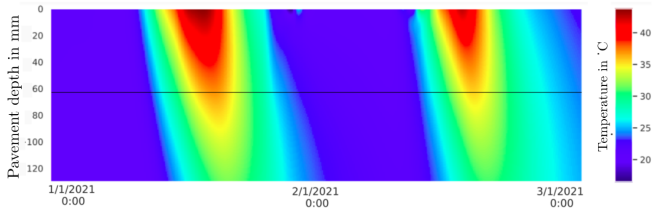

Figure 11. Simulated temperature distribution in asphalt layers (1 and 2) over a 48-hour period from 01.01.2021 to 03.01.2021.

5 Conclusions

This study demonstrates the development and application of a synthetic thermal digital twin for asphalt pavements using the Modelica language and the OpenModelica platform. The model captures transient heat conduction through a multilayer pavement structure by solving Fourier's heat equation with a finite difference scheme. Real-time synchronization between the physical and virtual twins is achieved through a PI controller that adjusts surface albedo to minimize temperature discrepancies.

The digital twin was applied to a case study in Costa Rica, where the model was validated using field temperature data collected over an 11-month period. The model showed good agreement with measured values, with an average absolute error of 1.08° C. The inclusion of rainfall and evaporation effects proved critical in accurately simulating surface cooling events, demonstrating the value of incorporating site-specific environmental conditions.

Results also highlighted the thermal behavior of asphalt layers under daily and seasonal cycles, revealing surface temperature peaks exceeding 50° C and internal thermal lags of about 1.5 hours. These conditions, typical in tropical climates, suggest that pavement distress mechanisms such as rutting and fatigue cracking may be more relevant than thermal cracking.

Finally, the modular architecture of the Modelica-based framework enables easy integration of additional environmental inputs and control strategies. This flexibility supports further development of comprehensive digital twins capable of coupling thermal and mechanical behavior for improved pavement performance prediction and infrastructure management.

In a real-world implementation, the Digital Twin would transmit predicted in-pavement temperature data from the virtual model to the physical road system. This information flow is essential for enabling proactive pavement management strategies. Forecasts of temperature at various depths can inform decisions such as optimal timing for de-icing, preventative maintenance to reduce thermal cracking, and adaptive load management during extreme heat or cold events. By delivering this predictive insight, the Virtual Twin supports data-driven interventions that can reduce operational costs, extend pavement lifespan, and enhance roadway safety. Although this study focuses on a synthetic case without direct communication to a physical system, the proposed architecture is designed to support such bidirectional data exchange in future deployments.

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