Model Based Control and Analysis of Gas Lifted Oil Field for Optimal Operation

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Abstract

This paper describes mathematical modeling, optimization, and analysis of a gas lift oil field with five wells. A global sensitivity analysis using the variance-based method is performed to classify the parameters, which are highly sensitive and uncertain simultaneously. An improved model is further used to design a model-based predictive controller to optimally distribute a limited supply of lift gas among the oil wells. Several simulation cases showed an increase in the total oil production, and all the constraints were fully satisfied when the deterministic NMPC was applied to the nominal model. The effect of parametric uncertainty is studied by applying the deterministic NMPC to the plant model containing the uncertain parameters. It has been shown that under the presence of uncertainty, robust constraint satisfaction is not guaranteed with some constraints not being satisfied, leading to unachievable and unrealistic lift gas distribution.

Keywords: Gas Lifted Oil Wells, Model Predictive Control, Global Sensitivity Analysis, Dynamic Modeling and Simulation, Parametric Uncertainty

1 Introduction

It is always of interest to manage and plan resources efficiently to obtain profit as much as possible from a given resource. In this sense, the oil and gas industry is not an exception. Hence, the optimal distribution of available gas is crucial to maximizing total oil production in a gas-lifted oil field where the multiple oil wells share the lift gas supplied by a common source.

In a gas lifted oil field, an artificial external mechanism is exploited to bring the dead wells back to life or increase the production rates from the naturally flowing wells. A continuous flow gas lifted oil field normally consists of multiple gas lift oil wells sharing lift gas from a common supply pipeline. A single gas lifted oil well is shown in Figure 1.In this system which is mostly used to extract the lighter crude oils, the high-pressurized natural gas is continuously injected into the annulus of the well through the gas lift choke valve. The injected gas finds its way into tubing at some points located at proper depths and mixes with the multiphase fluid from the reservoir. As a result of this mixing, the density of the fluid in the tubing will be reduced, which means that the flowing pressure losses in the tubing reduce. Consequently, the reservoir pressure will be able to overcome the flowing resistance in the well and push the reservoir fluid to the surface.

Each well has its own inflow characteristics. For example, two oil wells in the same field may produce different amount of oil even when the same amount of lift gas is injected into them. In other words, there is no rule of thumb on how to distribute the available lift gas among the oil wells to obtain the maximum possible oil production from the field. For optimal distribution of lift gas among the wells, model based real-time optimizer (RTO) can be used. For this an accurate mechanistic model of the process, which should be simultaneously simple enough to be used for real-time optimization and control purposes should be used.

Modeling and control of gas lifted oil field has been studied before in (Sharma et al., 2011), where some simplifying assumptions were made that may not reflect reality. For example, the fluid that comes out of the reservoir was assumed to be pure oil (without gas coming from the reservoir) and all the well parameters were assumed to be deterministic. This model had been used further in optimization of lift gas allocation as nonlinear optimization in (Sharma et al., 2012). This model has been improved in (Krishnamoorthy et al., 2016) by considering the gas to oil ratio. The long term production optimization under uncertainty has been studied in (Capolei et al., 2015; Hanssen et al., 2017) using economic MPC. But when it comes to the short-term optimization, most of the works either consider a deterministic model, which means they simply disregard uncertainty, or they limit the research scope to steady-state optimization using a very simplified linear model (Hanssen and Foss, 2015). Recently, a few papers have been published on real-time process optimization under the presence of uncertainty (Krishnamoorthy et al., 2019) to address the challenges in this area.

The first purpose of this paper is to improve the existing mathematical model of gas lifted oil fields with more realistic assumptions. To achieve this goal, the fluid that comes out of the reservoir is considered to be a mixture of oil, water, and gas. Furthermore, parametric uncertainties are considered for some parameters such as gas to oil ratio and productivity index. The second aim of the paper is to classify parameters that are both highly sensitive and uncertain simultaneously. Therefore, a global sensitivity analysis is performed to study how the uncertainty in output (total oil production from the field) can be apportioned to different sources of uncertainty in the model parameters. The first order and total-effect sensitivity indices are calculated using the variance-based method due to its valuable features, such as the inclusion of interaction effects among input factors (Saltelli et al., 2008). The third goal is to study the effect of parametric uncertainty on lift gas distribution optimization problem. Considering the operational constraints of the process and the inherent robustness (to a certain extent) of the receding horizon strategy, a deterministic nonlinear model based predictive controller is designed based on the nominal plant model to optimally distribute a limited supply of lift gas being shared to several oil wells in the field. Several simulation cases are performed to study the performance of the optimal controller under varying operational scenarios. Simulation results show that the total oil production will be increased and all the constraints will be satisfied when the deterministic NMPC is applied to the nominal model. The effect of parametric uncertainty is shown by applying the deterministic NMPC to the plant model containing uncertain parameters and it has been shown that some constraints will be violated which suggests that the uncertainties should be considered explicitly in the optimal control problem.

The rest of the paper is organized as follows. Section 2 describes mathematical modeling of the gas lifted oil field system, openloop simulation results and the sensitivity analysis. Standard nonlinear model predictive control design, simulation results and stochastic analysis are presented in Section 3 before concluding in Section 4.

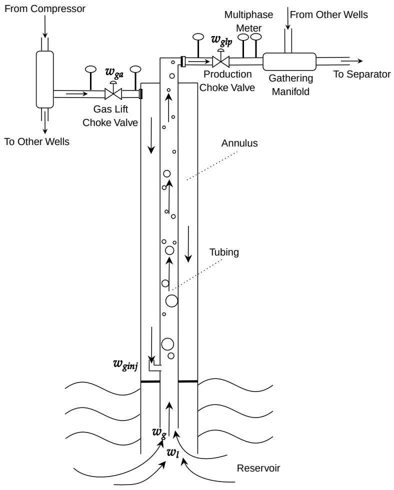

Figure 1. Schematic diagram of a single gas lift oil well

2 Modeling and Sensitivity Analysis

The considered gas lifted oil field of this paper consists of five oil wells that share a common gas distribution pipeline and common gathering manifold. A compressor discharges highly pressurized lift gas into the common gas distribution pipeline where it should be distributed among the oil wells. Considering a single oil well, the lift gas mass flow rate from the common distribution manifold into the well's annulus is denoted by w^i_{ga} where the superscript i refers to the i^{th} oil well. Then, the high pressure lift gas is injected from annulus into tubing (w^i_{ginj}) at a proper depth through the gas injection valve which is always open and only passes the flow in one direction. The injected gas mixes with the multiphase fluid (mixture of oil, water, and the gas from the reservoir) and reduces its density. This causes the hydrostatic pressure of the fluid column in tubing above the injection point and consequently the bottom hole pressure to drop. As a result, the differential pressure between the reservoir and the bottom hole pressure will increase and pushes the liquid column to flows upward to the surface. The produced mixture flows out through all the production choke valves (w^i_{glp}) is collected in the common gathering manifold and finally transported to the separator where they are separated into their corresponding compartments. The gas is then sent back to the compressor system and recycled to be used for lifting purposes.

Friction losses in the pipelines have not been taken into account since it might not be important for the sole purpose of control. All phases of the multiphase fluid are assumed to be evenly distributed with no slugging. The temperature of lift gas and the multiphase fluid is assumed to be constant at 280 K at all sections of the pipelines and the reservoir pressure is assumed to be constant at 150 bar. All the assumption are based on expert knowledge from Equinor ASA.

2.1 Model Description

The model is developed considering all the components of a typical gas lifted oil well as shown in Figure 1. The differential equations in model are obtained from the mass balances of each compartment. The algebraic equations are mostly density models, pressure models, flow models, and so on, which are obtained from equations of states, valve equations, and first principal modeling techniques.

Considering the mass of lift gas in annulus m^i_{ga}, mass of the gas in the tubing above the injection point m^i_{gt}, and mass of the liquid (mixture of oil and water) in the tubing above the injection point m^i_{lt} as three states and applying the mass balance, three corresponding differential equations are given by:

$$\dot{m}^i_{\text{ga}} = w^i_{\text{ga}} - w^i_{\text{ginj}} \tag{1}$$

$$\dot{m}^i_{\text{gt}} = w^i_{\text{ginj}} + w^i_{\text{g}} - w^i_{\text{gp}} \tag{2}$$

$$\dot{m}^i_{\text{lt}} = w^i_{\text{l}} - w^i_{\text{lp}} \tag{3}$$

w^i_{ga} is the mass flow rate of the injected lift gas into each well from the gas lift choke valve (system input), w^i_{ginj} is the mass flow rate of the gas injection from the annulus into the tubing, w^i_{gp} and w^i_{lp} are the produced gas and liquid phase mass flow rates from the production choke valve, respectively, and w^i_{g} and w^i_{l} are the gas and liquid mass flow rates from the reservoir into the well. w^i_{glp} is the total mass flow rate of all phases from the production choke valve and w^i_{op} is the oil compartment of the w^i_{lp}. All the flow equations are given by:

$$w^i_{\mathrm{ginj}} = K^i Y^i_2 \sqrt{\rho^i_{\mathrm{ga}} \max(P^i_{\mathrm{ainj}} - P^i_{\mathrm{tinj}}, 0)} \tag{4}$$

$$w^i_{\mathrm{gp}} = \frac{m^i_{\mathrm{gt}}}{m^i_{\mathrm{gt}} + m^i_{\mathrm{lt}}} w^i_{\mathrm{glp}} \tag{5}$$

$$w^i_{\mathrm{lp}} = \frac{m^i_{\mathrm{lt}}}{m^i_{\mathrm{gt}} + m^i_{\mathrm{lt}}} w^i_{\mathrm{glp}} \tag{6}$$

$$w^i_{\mathrm{l}} = PI^i \max(P_r - P^i_{\mathrm{wf}}) \tag{7}$$

$$w^i_{\mathrm{g}} = GOR^i w^i_{\mathrm{l}} \tag{8}$$

$$w^i_{\mathrm{glp}} = C_v(u^i_2) Y^i_3 \sqrt{\rho^i_{\mathrm{m}} \max(P^i_{\mathrm{wh}} - P_s, 0)} \tag{9}$$

$$w^i_{\mathrm{op}} = \frac{\rho_o}{\rho_w}(1 - WC^i) w^i_{\mathrm{lp}} \tag{10}$$

P^i_{a} is the pressure of lift gas in annulus downstream the gas lift choke valve, P^i_{ainj} is the pressure upstream the gas injection valve in the annulus and P^i_{tinj} is the pressure downstream the gas injection valve in the tubing, and P^i_{wh} and P^i_{wf} are the well head and bottom hole pressure respectively. All the pressures are given by:

$$P^i_{\mathrm{a}} = \frac{z m^i_{\mathrm{ga}} R T^i_{\mathrm{a}}}{M A^i_{\mathrm{a}} L^i_{\mathrm{a\_tl}}} \tag{11}$$

$$P^i_{\mathrm{ainj}} = P^i_{\mathrm{a}} + \frac{m^i_{\mathrm{ga}}}{A^i_{\mathrm{a}} L^i_{\mathrm{a\_tl}}} g L^i_{\mathrm{a\_vl}} \tag{12}$$

$$P^i_{\mathrm{tinj}} = \frac{z m^i_{\mathrm{gt}} R T^i_{\mathrm{t}}}{M V^i_G} + \frac{\rho^i_{\mathrm{m}} g L^i_{\mathrm{t\_vl}}}{2} \tag{13}$$

$$P^i_{\mathrm{wh}} = \frac{z m^i_{\mathrm{gt}} R T^i_{\mathrm{t}}}{M V^i_G} - \frac{\rho^i_{\mathrm{m}} g L^i_{\mathrm{t\_vl}}}{2} \tag{14}$$

$$P^i_{\mathrm{wf}} = P^i_{\mathrm{tinj}} + \rho^i_{\mathrm{l}} g L^i_{\mathrm{r\_vl}} \tag{15}$$

ρ^i_{ga} is the average density of gas in the annulus. ρ^i_{gl} is the density of liquid phase (oil and water mixture), ρ^i_{m} is the average density of multiphase mixture in tubing above the injection point. Y^i_2 and Y^i_3 are the gas expandability factor for the gas that passes through gas injection valve and production choke valve, respectively. V^i_G is the volume of gas present in the tubing above the gas injection point, and $C_v(u^i_2)$ is the production choke valve characteristics as its opening. All the densities and other variables are given by:

$$\rho^i_{\mathrm{ga}} = \frac{M(P^i_{\mathrm{a}} + P^i_{\mathrm{ainj}})}{2 z R T^i_{\mathrm{a}}} \tag{16}$$

$$\rho^i_{\mathrm{l}} = \rho_w WC^i + \rho_o(1 - WC^i) \tag{17}$$

$$\rho^i_{\mathrm{m}} = \frac{m^i_{\mathrm{gt}} + m^i_{\mathrm{lt}}}{A^i_{\mathrm{t}} L^i_{\mathrm{t\_tl}}} \tag{18}$$

$$Y^i_2 = 1 - \alpha_{\mathrm{Y}} \frac{P^i_{\mathrm{ainj}} - P^i_{\mathrm{tinj}}}{\max\left(P^i_{\mathrm{ainj}}, P^{\mathrm{min}}_{\mathrm{ainj}}\right)} \tag{19}$$

$$Y^i_3 = 1 - \alpha_{\mathrm{Y}} \frac{P^i_{\mathrm{wh}} - P_s}{\max\left(P^i_{\mathrm{wh}}, P^{\mathrm{min}}_{\mathrm{wh}}\right)} \tag{20}$$

$$V^i_G = A^i_{\mathrm{t}} L^i_{\mathrm{t\_tl}} - \frac{m^i_{\mathrm{lt}}}{\rho^i_{\mathrm{l}}} \tag{21}$$

$$C_v(u^i_2) = \begin{cases} 0 & \text{if } u^i_2 < 5 \\ 30.303 u^i_2 - 151.788 & \text{if } 5 < u^i_2 < 50 \\ 136.5 u^i_2 - 5460 & \text{if } 50 < u^i_2 \end{cases} \tag{22}$$

Note that the dynamic model 1 to 22 could be written as an explicit ODE (ordinary differential equations) by simply eliminating the algebraic variables. So the model in compact form is given by:

$$\dot{x} = f(x, u) \tag{23}$$

$$y_1 = h_1(x, u) \tag{24}$$

$$y_2 = h_2(x, u) \tag{25}$$

where x and u are the states and system inputs, and y_1 and y_2 are two desired outputs

$$x = \begin{bmatrix} m^1_{\mathrm{ga}} & \dots & m^5_{\mathrm{ga}} & m^1_{\mathrm{gt}} & \dots & m^5_{\mathrm{gt}} & m^1_{\mathrm{lt}} & \dots & m^5_{\mathrm{lt}} \end{bmatrix}^T \tag{26}$$

$$u = \begin{bmatrix} w^1_{\mathrm{ga}} & w^2_{\mathrm{ga}} & w^3_{\mathrm{ga}} & w^4_{\mathrm{ga}} & w^5_{\mathrm{ga}} \end{bmatrix}^T \tag{27}$$

$$y_1 = \sum_{i=1}^{5} w^i_{\mathrm{op}} \tag{28}$$

$$y_2 = \sum_{i=1}^{5} w^i_{\mathrm{glp}} \tag{29}$$

2.2 Uncertainties

In this work, the productivity index PI which is a mathematical means of expressing the reservoir's ability to deliver fluids to the wellbore, gas to oil ratio GOR which is defined as the mass ratio of produced gas to produced liquid (oil and water), and water cut WC which is defined as the volumetric flow rate of water to the total produced liquid, are considered to be constant but unknown parameters. Considering the five oil wells, there exist fifteen uncertain parameters in the system that makes it visually impossible to show the uncertainty region. Nevertheless, the uncertainty region of one well is shown in Figure 2 as an example. All the uncertain parameters of all the five wells in this paper are assumed to have the same ±20% deviation from their nominal values and uniform distribution. The reason of choosing uniform distribution is to challenge the controller.

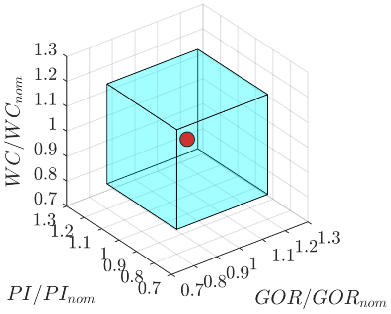

Figure 2. Unceratainty region with ±20% deviation.

2.3 Open Loop Simulation

The system is simulated in open loop using the nominal values of parameters provided in Table 1 and P_r and P_s are assumed to be 150 and 30 bar, respectively. The presented results in Figure 3 show that a decrease in the injected lift gas flow rates causes an increase in bottom hole pressures, and consequently, the oil production flows decrease. This means that the model is capable of showing all the necessary dynamics of gas lifted oil field and will be used further to perform sensitivity analysis and to design nonlinear model predictive control.

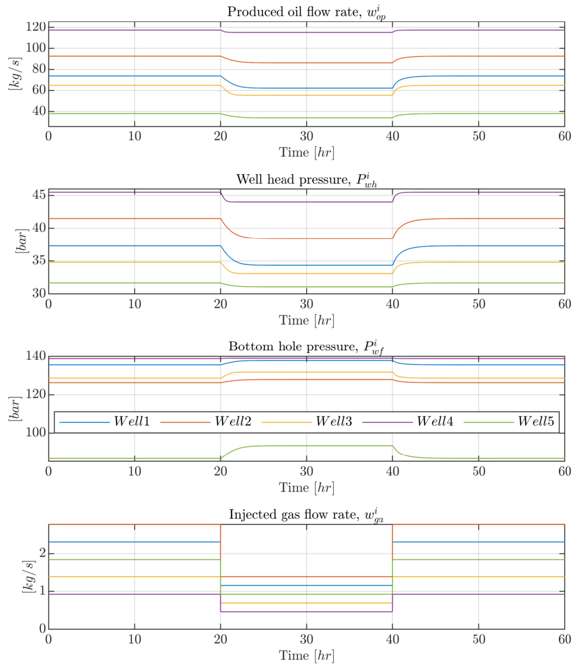

Figure 3. Open loop simulations of the nominal model

2.4 Global Sensitivity Analysis

It is useful to figure out which parameters have a strong/weak influence on the model output, especially under the presence of uncertainties, because the model based control design will be more problematic and needs more care if the uncertain parameters are sensitive as well. Variance based global sensitivity analysis method is selected due to its valuable features such as model independence, capacity to capture the influence of the full range of variation of each input factor, and appreciation of interaction effects among input factors.

The first order and total sensitivity indices are calculated using the variance based method introduced in (Saltelli et al., 2008) which is an improved extension of the original approach provided by (Sobol, 1993) and (Homma and Saltelli, 1996). Here only the results are presented and the readers are referred to the main reference for more information about the method due to the word limitation.

A number of 136000 Monte Carlo simulations have been done to calculate the sensitivity indices. Both sensitivity indices presented in Figure 4 show that for the considered uncertainty region introduced in Figure 2, gas to oil ratio is the most sensitive/influential parameter and productivity index and water cut are at the second and third place, respectively. In other words, the standard controller based on the nominal model will be more robust to deviation in water cut. On the other hand, a slight deviation in the gas to oil ratio leads to a severe mismatch between the nominal and uncertain model, therefore, poor performance is expected. These interpretations will be verified by stochastic analysis results in the following section.

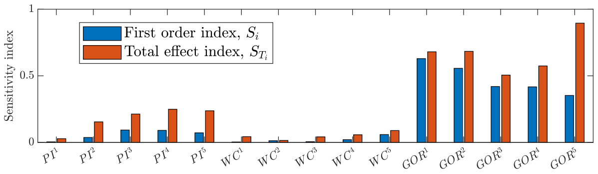

Figure 4. Sensitivity indices

3 Standard NMPC and Stochastic Analysis

3.1 Design of deterministic standard NMPC

The primary control objective is to maximize the total oil production of the field (output y_1) by manipulating the injected lift gas (u). Additionally, u and Δu are introduced to penalize excessive lift-gas utilization and large fluctuations in the control signals. Apart from the model equations, which obviously should be satisfied, the process is subjected to operational constraints. For example, the total injected lift gas should be equal to or less than the total available lift gas ($W_{\mathrm{gc}}^{\max}$) and the total produced fluid should not exceed the maximum capacity of the separator ($W_{\mathrm{s}}^{\max}$). There are also upper and lower bounds on control

Table 1. Nominal values of well parameters used for simulation.

Parameter	*Well1*	*Well2*	*Well3*	*Well4*	*Well5*	*Unit*
K	68.43	67.82	67.82	69.26	66.22	$[\frac{\sqrt{\frac{\text{kgm}^3}{\text{bar}}}}{\text{hr}}]$
$PI(1.0\text{e}{+}4)$	2.51	1.63	1.62	4.75	0.232	$\frac{\text{kg/hr}}{\text{bar}}$
GOR	0.05	0.07	0.03	0.04	0.06	[kg/kg]
WC	0.20	0.10	0.25	0.15	0.05	$[\text{m}^3/\text{m}^3]$
$L_{\text{a\_tl}}/L_{\text{t\_tl}}$	2758	2559	2677	2382	2454	[m]
$L_{\text{a\_vl}}/L_{\text{t\_vl}}$	2271	2344	1863	1793	1789	[m]
A_{a}	0.0174	0.0174	0.0174	0.0174	0.0174	$[\text{m}^2]$
A_{t}	0.0194	0.0194	0.0194	0.0194	0.0194	$[\text{m}^2]$
$L_{\text{r\_vl}}$	114	67	61	97	146	[m]

inputs and change of control due to the physical limitation of the actuators (valves). Therefore, the optimal control problem formulation is given by:

$$\min_{x,u} \sum_{k=0}^{N-1} \left(-Q(y_{1,k})^2 + R\sum_{i=1}^{5} u_k^{i^2} + S\sum_{i=1}^{5} \Delta u_k^{i^2} \right) \quad (30)$$

$$\text{s.t.} \quad x_{k+1} = f(x_k, u_k, \theta_k) \quad (31)$$

$$\sum_{i=1}^{5} u_k^i \leq W_{\text{gc,k}}^{\max} \quad (32)$$

$$y_{2,k} \leq W_s^{\max} \quad (33)$$

$$u_{\text{LB}} \leq u_k^i \leq u_{\text{UB}} \quad (34)$$

$$\Delta u_{\text{LB}} \leq \Delta u_k^i \leq \Delta u_{\text{UB}} \quad (35)$$

where Q, R, and S are tuning weights and are chosen to be 1, 0.5, and 50, respectively. The total available lift gas $W_{\text{gc}}^{\max} = 9.22$[kg/s] and the maximum capacity of the separator $W_{\text{s}}^{\max} = 520$[kg/s]. The lower and upper bounds on the control signal are 0.323 and 11.66[kg/s]. Change of control also is limited between ± 0.15[kg/s]. A sampling time of 10 seconds and a prediction horizon of 25 timesteps (4.1 min) is used. These values are maintained constant throughout this paper.

3.2 Stochastic analysis of parametric uncertainty

In the first scenario, the deterministic NMPC was applied to the nominal model. As shown in Figure 5, open loop simulation started within the feasible region and the controller activated after 1 hour. The simulation results show a 12% increased in the total oil production from the field while all the constraints on the total available lift gas, capacity of separator and actuator limitations are fully satisfied.

In the other scenarios, the same controller is applied to the models containing uncertainties to see whether the controller can cope with the uncertainties in the model. For the extreme cases where the uncertain parameters take their maximums and minimums in the uncertainty region, severe oscillations were observed that led to instability.

Figure 6 shows the result of applying the nominal controller to the plant that only has -10% deviations in water cut, while the gas to oil ratio and productivity index are equal to their nominal values. It can be seen that the total oil production has been increased while the constraint on the maximum capacity of separator is violated. Although this case is not practically implementable, it worth to be noted that the same, or even smaller deviation (about 4%) in gas to oil ratio and productivity index leads to instability. This observation is consistent with the outcome from the sensitivity analysis that says the model is less sensitive to water cut rather than either gas to oil ratio or productivity index.

Figure 7 is the last scenario with -5, 8, and -2 percent deviations in productivity index, water cut, and gas to oil ratio, respectively, from their nominal values. The mismatch between the nominal and uncertain models can be observed from the total fluid production graph. In essence, it can be concluded that the deterministic NMPC is not sufficient for the gas lifted oil field model with uncertain parameters.

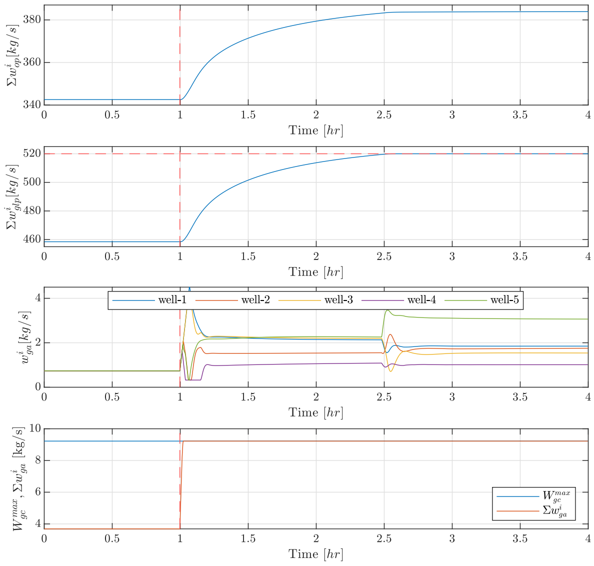

Figure 5. Performance of standard NMPC when it is applied to the nominal model.

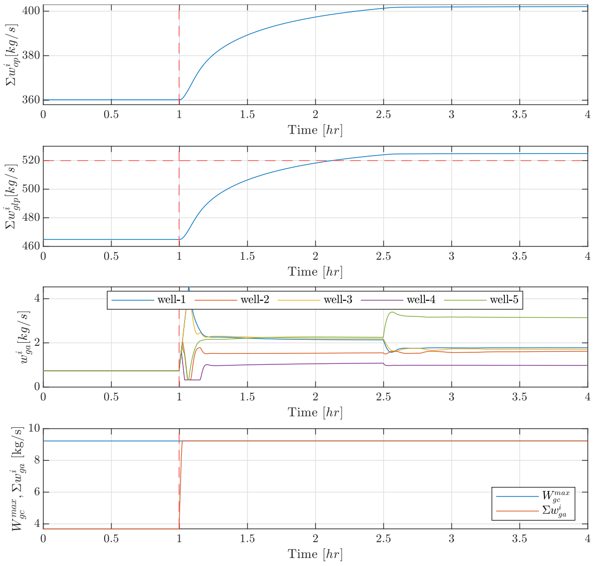

Figure 6. Performance of standard NMPC when it is applied to the uncertain model.

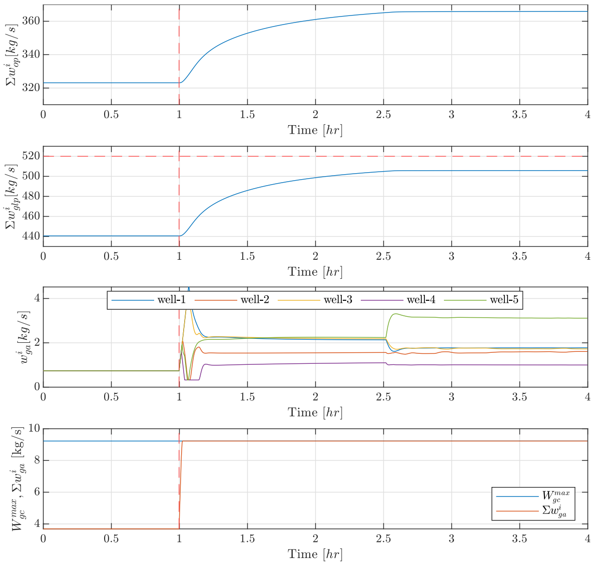

Figure 7. Performance of standard NMPC when it is applied to the uncertain model.

4 Conclusion

This paper presented a modeling framework for the gas lifted well system and total oil production maximization as a dynamic optimization control problem. The simulation results showed that the deterministic NMPC based on nominal model is capable of maximizing the total oil production of the nominal model while fulfills all the operational constraints subjected to the process; however, when the deterministic NMPC is applied to the model contains uncertainties, simulation results showed some constraints violations. This means that a deterministic NMPC is not sufficient to handle parametric uncertainties for this problem. Feasibility issues showed that the uncertainties need to be considered explicitly inside the optimization problem using robust or stochastic model predictive control. The future work includes using such advanced control methods to maximize total oil production while ensuring robust constraint satisfaction for all possible values of the uncertainties.

Acknowledgments

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References

Andrea Capolei, Bjarne Foss, and John B. Jørgensen. Profit and risk measures in oil production optimization. *IFAC-PapersOnLine*, 48(6):214–220, 2015. doi:10.1016/j.ifacol.2015.08.034.

Kristian G. Hanssen and Bjarne Foss. Production optimization under uncertainty - applied to petroleum production. *IFAC-PapersOnLine*, 48(8):217–222, 2015. doi:10.1016/j.ifacol.2015.08.184.

Kristian G. Hanssen, Andrés Codas, and Bjarne Foss. Closed-loop predictions in reservoir management under uncertainty. *SPE Journal*, 22(5):1585–1595, 2017. doi:10.2118/185956-PA.

Toshimitsu Homma and Andrea Saltelli. Importance measures in global sensitivity analysis of nonlinear models. *Reliability Engineering & System Safety*, 52(1):1–17, 1996. doi:10.1016/0951-8320(96)00002-6.

Dinesh Krishnamoorthy, Bjarne Foss, and Sigurd Skogestad. Real-time optimization under uncertainty applied to a gas lifted well network. *Processes*, 52(4), 2016. doi:10.3390/pr4040052.

Dinesh Krishnamoorthy, Sigurd Skogestad, and Johannes Jäschke. Multistage model predictive control with online scenario tree update using recursive bayesian weighting. *18th European Control Conference (ECC)*, pages 1443–1448, 2019. doi:10.23919/ECC.2019.8795839.

Andrea Saltelli, Marco Ratto, Terry Andres, Francesca Campolongo, Jessica Cariboni, Debora Gatelli, Michaela Saisana, and Stefano Tarantola. *Global Sensitivity Analysis: The Primer*. John Wiley & Sons, 2008. ISBN 978-0-470-72517-7.

Roshan Sharma, Kjetil Fjalestad, and Bjørn Glemmestad. Modeling and control of gas lifted oil field with five oil wells. *the 52nd International Conference of Scandinavian Simulation Society*, pages 47–59, 2011.

Roshan Sharma, Kjetil Fjalestad, and Bjørn Glemmestad. Optimization of lift gas allocation in a gas lifted oil field as nonlinear optimization problem. *Modeling, Identification and Control*, 33(1):13–25, 2012.

Ilya M. Sobol. Sensitivity analysis for non-linear mathematical models. *Mathematical modelling and computational experiment*, 1:407–414, 1993.