ESP Lifted Oil Field: Core Model, and Comparison of Simulation Tools

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Abstract

Optimal operation of petroleum production is important in a transition from energy systems based on fossil fuel to sustainable systems. One sub-process in petroleum production deals with transport from the (subsea) well-bore to a topside separator. Here, a simple model in Sharma & Glemmestand (2014) has been streamlined into a dynamic model suitable for illustration of the dynamics of oil transport, as well for control studies. The advantages of using dimensionless equipment models are emphasized. The model is then used to compare two popular modeling languages: Modelica, and ModelingToolkit for Julia. Key advantages and disadvantages of these two languages are emphasized.

1 Introduction

1.1 Background

Petroleum products have been key energy carriers for more than a century. Current focus on climate[1] implies a change towards sustainable energy carriers. To succeed in this change, a transition period from the use of fossil fuel is necessary. In the transition, improved operation of petroleum production through model based optimal operation will be necessary. Petroleum production entails slow (reservoir; months) and fast (reservoir-to-separator; seconds) subsystems; a focus of research project "DigiWell"[2]. Vertical transport of petroleum from oil well to surface requires sufficient pressure to counteract gravitational and friction forces. If the oil-well heel pressure is insufficient for such transport, either (i) gas is injected in the vertical pipe to "blow" the petroleum fluids to the surface, or (ii) an electrical submersed pump [ESP] is installed in the vertical pipe to sufficiently increase the pressure. Here, we study the dynamics of transport from the reservoir formation to a surface manifold via an ESP, and further horizontal transport from the manifold to a separator.

Industrial simulation tools typically put main emphasis on the dynamics of the *reservoir* (time constant: months) and use steady state models for the reservoir-to-surface transport. This emphasis is inadequate for daily operation and control. Here, a simple dynamic reference model for oil transport from reservoir to separator is provided. The model provides an understanding of the dynamic behavior of such systems, and is suitable for industrial control design. Emphasis is put on a simple, yet stringent model development, while avoiding unit complexities.

1.2 Previous work

Sharma & Glemmestand (2014) (Sharma, 2014) provide a dynamic model of oil transport from reservoir to separator suitable for control design; this model is the focus here. Binder et al. (2015) discuss an older model; other models typically are CFD models, etc., too complex for control design. Sharma's model considers a case with 4 vertical pipes from oil reservoirs to a single manifold, with 2 horizontal pipes from the manifold to a single separator. Each vertical pipe has an ESP, plus a choke valve at the manifold entrance; the pump speeds can be manipulated individually. The horizontal pipes have booster pumps to counteract friction losses. The original ESP model includes induction motors, but the dynamics of the pump actuator is fast, and is neglected in later work. Sharma & Glemmestand (2014) provide a novel ESP model, a simple model for a booster pump, and use a valve model based on on the ANSI/ISA S75.01 standard[3].

The model with ESP in Sharma (2014) is mainly relevant for the production of heavy oil. Several papers use this model in advanced industrial control studies Krishnamoorthy et al. (2016); Santana et al. (2021).

Mixtures of liquid oil and water form an emulsion when stirred (e.g., in a multi-stage ESP); for such emulsions, the viscosity — and hence the friction — varies dramatically with water content, Justiniano & Romero (2021). Sharma & Glemmestand (2014) as-

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[1] https://sdgs.un.org/goals

[2] See Acknowledgments.

[3] `http://integrated.cc/cse/ISA_750101_SPBd.pdf`

sume an unrealistic linear dependence of water fraction.

1.3 Structure of paper

Section 2 gives an overview of the transport system from oil reservoir via manifold to a separator, and key equipment models. Section 3 develops a simple mechanistic model of the system. Section 4 contrasts two modeling languages for simulation: Modelica and Julia's ModelingToolkit. Section 5 illustrates model behavior and the use of modeling/simulation tools. Finally, Section 6 provides some conclusions.

2 System description

We consider production of a mixture of water and crude oil in liquid phase.

2.1 System topology

Oil production *systems* merge several boreholes from the same or different reservoirs through vertical pipes into a manifold. Normally, more than one horizontal transport pipe are needed from the manifold to a separator for sufficient transport capacity. Water is commonly added to the manifold to reduce friction loss in the horizontal pipes. Figure 1 shows a system with n_{w} wells/vertical pipes and n_{t} transportation/horizontal pipes to the separator.

All vertical pipes are assumed connected to the same manifold pressure p_{m}; hence effluent *choke pressure* satisfies $p_{\mathrm{c}}^{\mathrm{e},j} = p_{\mathrm{c}}^{\mathrm{e}} = p_{\mathrm{m}}$ for all j. Likewise, all transport pipes end up in the same separator: $p_{\mathrm{s}}^{-,j} = p_{\mathrm{s}}$ for all j.

2.2 Fluid properties

The petroleum fluid properties are important. Density varies with pressure and temperature, $\rho\left(p,T\right)$. Neglecting temperature dependence, and assuming constant *isothermal compressibility*,

$$\rho = \rho_0 \exp\left(\beta_T \left(p - p_0\right)\right) \tag{1}$$

where $\left(\rho_0, p_0\right)$ is some reference state, and β_T is the (assumed) constant isothermal compressibility.[4]

Defining water cut χ_{w} as $\chi_{\mathrm{w}} \triangleq \dot{V}_{\mathrm{w}}/\dot{V}$: volumetric flow rate of water divided by total flow rate of the fluid, total density ρ becomes

$$\rho = \chi_{\mathrm{w}}\rho_{\mathrm{w}} + \left(1 - \chi_{\mathrm{w}}\right)\rho_{\mathrm{o}}; \tag{2}$$

here, ρ_{w} and ρ_{o} are constant densities of pure water and crude oil, respectively.

In reality, water and crude oil have different isothermal compressibilities. Here, we simplify and assume an overall value for β_{T}. Using data in Appendix 1, density ρ varies ca. $10\,\mathrm{kg/m^3}$ with pressure variation in the range 25–225 bar; we thus assume constant density in pipes, but a pressure-dependent density will be assumed in the manifold.

Sharma & Glemmestand (2014) propose a simple linear mixing rule for *kinematic viscosity* ν:

$$\nu = \chi_{\mathrm{w}}\nu_{\mathrm{w}} + \left(1 - \chi_{\mathrm{w}}\right)\nu_{\mathrm{o}}. \tag{3}$$

With ν known, *dynamic* viscosity μ can be computed (if needed) as

$$\mu = \nu\rho. \tag{4}$$

This linear interpolation model is used here, even though it is not physically realistic.

2.3 Well-bore production

Total production from the reservoir (formation pressure p_{f}) relates volumetric petroleum fluid rate $\dot{V}_{\mathrm{h}}$ at the well-bore heel as $\dot{V}_{\mathrm{h}} \propto p_{\mathrm{f}} - p_{\mathrm{h}}$, where p_{h} is heel pressure and the proportionality constant is the *productivity index*, which is unit-dependent. Here, we propose a dimensionless form instead,

$$\dot{V}_{\mathrm{h}} = \dot{V}_{\mathrm{pi}} \frac{p_{\mathrm{f}} - p_{\mathrm{h}}}{p_{\mathrm{pi}}^{\varsigma}} \tag{5}$$

where $\dot{V}_{\mathrm{pi}}$ is the productivity *capacity* in the same unit as $\dot{V}_{\mathrm{h}}$ and a scaling pressure $p_{\mathrm{pi}}^{\varsigma}$ which has the same unit as $p_{\mathrm{f}}, p_{\mathrm{h}}$.

2.4 Pump models

Pump models are typically given as

$$\Delta p_{\mathrm{p}} = \rho g h_{\mathrm{p}}; \tag{6}$$

pump *head* $h_{\mathrm{p}} = h_{\mathrm{p}}\left(\dot{V}, f_{\mathrm{p}}\right)$ with control input f_{p} — rotational pump frequency Hz, and volumetric flow rate $\dot{V}$.

Sharma & Glemmestand (2014) give a comprehensive model for the pump head of a *multi-stage ESP*. To ease change of units, their model is here rewritten in dimensionless form

$$\begin{aligned} \frac{h_{\mathrm{p}}\left(\dot{V}, f_{\mathrm{p}}\right)}{h_{\mathrm{p}}^{\varsigma}} &= \left(\frac{f_{\mathrm{p}}}{f_{\mathrm{p},0}}\right)^2 + a_1 \frac{f_{\mathrm{p}}}{f_{\mathrm{p},0}} \frac{\dot{V}}{\dot{V}^{\varsigma}} \\ &\quad + a_2 \left(\frac{\dot{V}}{\dot{V}^{\varsigma}}\right)^2 + a_3 \frac{f_{\mathrm{p},0}}{f_{\mathrm{p}}} \left(\frac{\dot{V}}{\dot{V}^{\varsigma}}\right)^3. \end{aligned} \tag{7}$$

In Eq. 7, $h_{\mathrm{p}}^{\varsigma}$ is a scaling head, f_{p} is the pump rotational frequency in the same unit as that of the nominal rotational frequency $f_{\mathrm{p},0}$, $\dot{V}$ is the actual volumetric flow rate out of the pump, $\dot{V}^{\varsigma}$ a scaling flow rate, and $a_1, \ldots, a_3$ are dimensionless model parameters[5].

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[4] Isothermal compressibility is the inverse of bulk modulus.

[5] Here, a_j is dimensionless, while in Sharma (2014) his parameters a_j have dimensions. This implies that the values of a_j here are different from those of a_j in Sharma (2014).

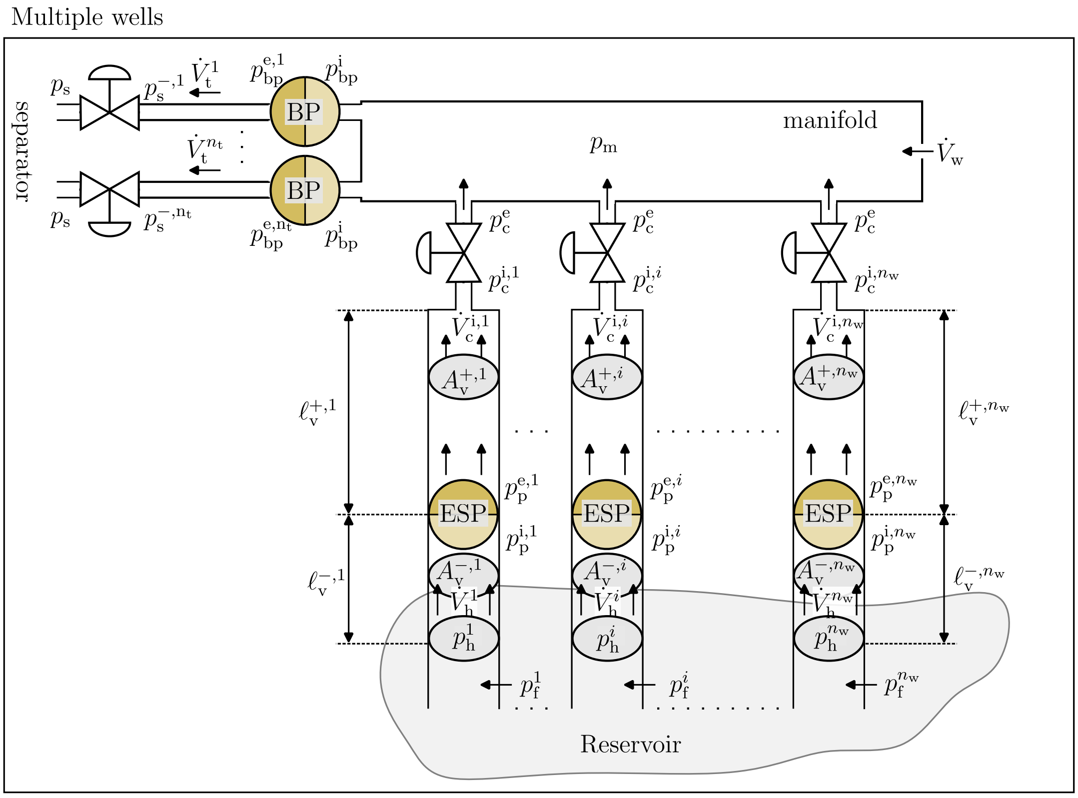

Figure 1. Multiple well system with n_w wells — possibly coming from different reservoirs, and n_t transport pipes to the separator; based on Sharma & Glemmestand (2014).

For the *booster pump* in the horizontal pipes, a simpler model is suggested in Sharma & Glemmestand (2014), here rewritten in dimensionless form as

$$\frac{\Delta p_{bp}\left(f_{bp}\right)}{\Delta p_{bp}^{\varsigma}} = \left(\frac{f_{bp}}{f_{bp,0}}\right)^2 \tag{8}$$

Here, $\Delta p_{bp}\left(f_{bp}\right)$ is the pressure increase at the given pump frequency/speed f_{bp}, in the same unit as $\Delta p_{bp}^{\varsigma}$ — which is the pressure increase at the nominal pump frequency $f_{bp,0}$.

2.5 Valve models

Sharma & Glemmestand (2014) base their valve models on the ANSI/ISA S75.01 standard[6]. Here, we instead propose a dimensionless description with extension to a control valve as

$$\dot{m} = \dot{m}_v^c \cdot f\left(u_v\right) \frac{\rho_i}{\rho_e} \sqrt{\frac{\left(p_i - p_e\right)/p^{\varsigma}}{\rho_i/\rho^{\varsigma}}} \tag{9}$$

where $\dot{m}_v^c$ is the valve mass flow rate capacity, $u_v \in [0,1]$ is the valve control signal, $f : [0,1] \to [0,1]$ is the valve characteristics, ρ_i, ρ_e are influent and effluent densities, respectively, p_i, p_e are influent and effluent pressures, respectively, while $\rho^{\varsigma}, p^{\varsigma}$ are scaling density and pressure, respectively.

2.6 Friction loss

The friction drop along the pipe can be given by the Darcy-Weisbach equation,

$$\frac{\Delta p_f}{\ell} = f_D \frac{\rho}{2} \frac{v^2}{D} \tag{10}$$

where f_D is Darcy's friction factor, given by Colebrook's[7] implicit expression. One explicit approximation to Colebrook's expression is due to Swamee and Jain (Brkić, 2011),

$$\frac{1}{\sqrt{f_D}} = -2 \cdot \log_{10}\left(\frac{5.74}{N_{Re}^{0.9}} + \frac{\varepsilon/D}{3.7}\right), \tag{11}$$

where N_{Re} is the Reynolds number,

$$N_{Re} = \frac{\rho v D}{\mu} = \frac{vD}{\nu}, \tag{12}$$

μ is *dynamic* viscosity, ν is *kinematic* viscosity, and ε is the "roughness height" of the pipe internals. Linear velocity v is related to volumetric flow rate $\dot{V}$ by

$$\dot{V} = vA \tag{13}$$

where A is the cross-sectional area of the pipe.

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[6] `http://integrated.cc/cse/ISA_750101_SPBd.pdf`

[7] The Colebrook equation, or sometimes known as the Colebrook-White equation.

2.7 *Why dimensionless models?*

As an example, consider the ESP model in Eq. 7. In the original formulation in Sharma (2014), the volumetric flow-rate is hard-coded to use a given unit for the flow rate, e.g., $\mathrm{m}^3/\mathrm{day}$. If the dynamic model requires the flow rate in other units for dimensional consistency, it may take considerable work to re-compute the polynomial coefficients to achieve this. In summary: use of dimensionless models simplifies the process of changing units, and reduces the chance of introducing errors.

3 Dynamic model

3.1 *Balance laws*

The model is based on the total mass balance (manifold) and the linear momentum balance (pipes). The total mass balance is expressed as

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \dot{m}_\mathrm{i} - \dot{m}_\mathrm{e} \tag{14}$$

where m is accumulated mass in the system, t is time, $\dot{m}$ is mass flow rate, and indices i, e denote influent and effluent, respectively.
The linear momentum balance is

$$\frac{\mathrm{d}\mathfrak{m}}{\mathrm{d}t} = \dot{\mathfrak{m}}_\mathrm{i} - \dot{\mathfrak{m}}_\mathrm{e} + F, \tag{15}$$

where $\mathfrak{m}$ is linear momentum given as $\mathfrak{m} = mv$ with linear velocity v, $\dot{\mathfrak{m}}$ is momentum flow rate given as $\dot{\mathfrak{m}} = \dot{m}v$, and F is total force. With constant fluid density, $\dot{\mathfrak{m}}_\mathrm{i} = \dot{\mathfrak{m}}_\mathrm{e}$, and the momentum balance reduces to Newton's law, $\frac{\mathrm{d}\mathfrak{m}}{\mathrm{d}t} = F$.

3.2 *Vertical pipes with ESP*

We assume constant density in the pipes, hence Eq. 15 reduces to Newton's law. Momentum is given as $\mathfrak{m} = mv$ with $m = \rho \dot{V}$, and v related to $\dot{V}$ by Eq. 13. The total force is $F = F_\mathrm{p} + F_\mathrm{b} - F_\mathrm{f} - F_\mathrm{g}$, with

- Pressure forces at inlet and outlet of the pipe,

$$F_\mathrm{p} = p_\mathrm{h}A - p_\mathrm{c}^\mathrm{i}A \tag{16}$$

- Possible pressure boost due to a pump,

$$F_\mathrm{b} = \Delta p_\mathrm{p} A, \tag{17}$$

  with Δp_p given by Eqs. 6, 7,

- Friction loss,

$$F_\mathrm{f} = \Delta p_\mathrm{f} A, \tag{18}$$

  with Δp_f given by Eqs. 10, 11, 12, 13,

- Flow against gravity, with a lift height h,

$$F_\mathrm{g} = \Delta p_\mathrm{g} A, \tag{19}$$

  with

$$\Delta p_\mathrm{g} = \rho_\mathrm{v} g h.$$

In addition, we need information about how flow rate $\dot{V}$ relates to the bottom hole pressure via the productivity capacity, Eq. 5.
The most structured formulation would be to pose the momentum balance (here: Newton's law) as the differential equation, and add all necessary algebraic equations. However, the OpenModelica DAE solver struggles with such a formulation: the valve equation Eq. 9 is implicit in pressure difference; in the iteration to find $\Delta p_\mathrm{v} = p_\mathrm{i} - p_\mathrm{e}$, if Δp_v becomes negative, the square root gives a complex number, and the simulation crashes. Instead, we have changed the differential variable to $\dot{V}$; then the valve equation can be inverted and expressed as $\Delta p_\mathrm{v} \propto \dot{V}^2$.
The following formulation is used in OpenModelica and ModelingToolkit:

$$\frac{\mathrm{d}\dot{V}_\mathrm{v}}{\mathrm{d}t} = \frac{p_\mathrm{h} - p_\mathrm{i}^\mathrm{c} + \Delta p_\mathrm{p} - \Delta p_\mathrm{f} - \Delta p_\mathrm{g}}{\rho_\mathrm{v}\ell/A_\mathrm{v}} \tag{20}$$

$$\rho_\beta^0 = \chi_\mathrm{w}\rho_\mathrm{w} + (1-\chi_\mathrm{w})\,\rho_\mathrm{o} \tag{21}$$

$$\nu = \chi_\mathrm{w}\nu_\mathrm{w} + (1-\chi_\mathrm{w})\,\nu_\mathrm{w} \tag{22}$$

$$\mu = \rho_\beta^0 \nu \tag{23}$$

$$\rho_\mathrm{v} = \rho_\beta^0 \exp\left(\beta_T\left(p_\mathrm{c}^\mathrm{i} - p_\beta^0\right)\right) \tag{24}$$

$$p_\mathrm{h} = p_\mathrm{f} - p_\mathrm{pi}^\varsigma \frac{\dot{V}_\mathrm{v}}{\dot{V}_\mathrm{pi}} \tag{25}$$

$$\dot{m}_\mathrm{v} = \rho_\mathrm{v}\dot{V}_\mathrm{v} \tag{26}$$

$$p_\mathrm{i}^\mathrm{c} = p_\mathrm{m} + p_\mathrm{v}^\varsigma \frac{\rho_\mathrm{v}}{\rho_\mathrm{v}^\varsigma}\left(\frac{\dot{m}_\mathrm{v}}{\dot{m}_\mathrm{v}^\mathrm{c}}\right)^2 \tag{27}$$

$$\begin{aligned} h_\mathrm{p} = h_\mathrm{p}^\varsigma \Bigg(\left(\frac{f_\mathrm{p}}{f_{\mathrm{p},0}}\right)^2 + a_1 \frac{f_\mathrm{p}}{f_{\mathrm{p},0}} \frac{\dot{V}}{\dot{V}^\varsigma} \\ + a_2 \left(\frac{\dot{V}}{\dot{V}^\varsigma}\right)^2 + a_3 \frac{f_{\mathrm{p},0}}{f_\mathrm{p}} \left(\frac{\dot{V}}{\dot{V}^\varsigma}\right)^3 \Bigg) \end{aligned} \tag{28}$$

$$\Delta p_\mathrm{p} = \rho_\mathrm{v} g h_\mathrm{p} \tag{29}$$

$$v_\mathrm{v} = \frac{\dot{V}_\mathrm{v}}{A} \tag{30}$$

$$N_\mathrm{Re} = \frac{\rho_\mathrm{v} v_\mathrm{v} d_\mathrm{v}}{\nu_\mathrm{v}} \tag{31}$$

$$f_\mathrm{D}^\mathrm{v} = \frac{1}{4\left(\log_{10}\left(\frac{5.74}{N_\mathrm{Re}^{0.9}} + \frac{\varepsilon_\mathrm{v}/d_\mathrm{v}}{3.7}\right)\right)^2} \tag{32}$$

$$\Delta p_{\mathrm{f}} = \ell \cdot f_{\mathrm{D}}^{\mathrm{v}} \frac{\rho_{\mathrm{v}}}{2} \frac{v_{\mathrm{v}}^2}{d_{\mathrm{v}}} \tag{33}$$

$$\Delta p_{\mathrm{g}} = \rho_{\mathrm{v}} g h. \tag{34}$$

If we only consider the model of a single vertical pipe, we need to specify (i) initial state (i.e., $\dot{V}_{\mathrm{v}}$), (ii) all "input" variables, i.e., p_{f}, f_{p}, p_{m}, and possibly water cut χ_{w}, and (iii) all parameters, i.e., ρ_{w}, ρ_{o}, ν_{w}, ν_{o}, p_{β}^{0}, ℓ, A, $p_{\mathrm{pi}}^{\varsigma}$, $\dot{V}_{\mathrm{pi}}$, $p_{\mathrm{v}}^{\varsigma}$, $\rho_{\mathrm{v}}^{\varsigma}$, $\dot{m}_{\mathrm{v}}^{\mathrm{c}}$, $h_{\mathrm{p}}^{\varsigma}$, $f_{\mathrm{p},0}$, $\dot{V}^{\varsigma}$, a_1,a_2, a_3, g, d_{v}, ν_{v}, ε_{v}, h.

3.3 *Manifold*

We assume a perfectly mixed manifold. Assuming constant manifold volume V_{m}, and adding water at flow rate $\dot{V}_{\mathrm{w}}$ to dilute the fluid to manifold water cut $\chi_{\mathrm{w}}^{\mathrm{m}}$, thus reducing friction loss in the pipe towards separator, $\dot{V}_{\mathrm{w}}$ must be approximately

$$\dot{V}_{\mathrm{w}} = \frac{\chi_{\mathrm{w}}^{\mathrm{m}} - \chi_{\mathrm{w}}}{1 - \chi_{\mathrm{w}}^{\mathrm{m}}} \dot{V}_{\mathrm{v}}. \tag{35}$$

Total mass balance for the manifold can then be expressed as

$$\frac{\mathrm{d}p_{\mathrm{m}}}{\mathrm{d}t} = \frac{1}{\rho_{\mathrm{m}} V_{\mathrm{m}} \beta_T} \left(\rho_{\mathrm{v}} \dot{V}_{\mathrm{v}} + \rho_{\mathrm{w}} \dot{V}_{\mathrm{w}} - \rho_{\mathrm{m}} \dot{V}_{\mathrm{t}}\right) \tag{36}$$

$$\rho_{\beta}^{0} = \chi_{\mathrm{w}}^{\mathrm{m}} \rho_{\mathrm{w}} + (1 - \chi_{\mathrm{w}}^{\mathrm{m}}) \rho_{\mathrm{o}} \tag{37}$$

$$\rho_{\mathrm{m}} = \rho_{\beta}^{0} \exp\left(\beta_T \left(p_{\mathrm{m}} - p_{\beta}^{0}\right)\right) \tag{38}$$

$$\dot{V}_{\mathrm{w}} = \frac{\chi_{\mathrm{w}}^{\mathrm{m}} - \chi_{\mathrm{w}}}{1 - \chi_{\mathrm{w}}^{\mathrm{m}}} \dot{V}_{\mathrm{c}}^{\mathrm{i}} \tag{39}$$

In practice, a control system must be used to manipulate $\dot{V}_{\mathrm{w}}$ instead of using Eq. 35.

For the manifold model, we must know (i) the initial manifold pressure, (ii) the vertical inflow $\dot{V}_{\mathrm{v}}$ and the horizontal transport flow $\dot{V}_{\mathrm{t}}$ from manifold to separator, as well as manifold water cut $\chi_{\mathrm{w}}^{\mathrm{m}}$, and (iii) parameters.

3.4 *Transport pipe with booster pump*

The model of the horizontal pipe from manifold to separator is almost identical to the vertical pipe from reservoir to manifold. The essential differences are (i) no gravity pressure drop, (ii) simpler booster pump model, (iii) neglecting pressure drop from pipe into separator, (iv) no need for a production capacity model. The complete model is

$$\frac{\mathrm{d}\dot{V}_{\mathrm{t}}}{\mathrm{d}t} = \frac{p_{\mathrm{m}} - p_{\mathrm{s}} + \Delta p_{\mathrm{bp}} - \Delta p_{\mathrm{f}}^{\mathrm{t}}}{\rho_{\mathrm{t}} \ell_{\mathrm{t}} / A_{\mathrm{t}}} \tag{40}$$

$$\rho_{\beta}^{0,\mathrm{t}} = \chi_{\mathrm{w}}^{\mathrm{m}} \rho_{\mathrm{w}} + (1 - \chi_{\mathrm{w}}^{\mathrm{m}}) \rho_{\mathrm{o}} \tag{41}$$

$$\nu_{\mathrm{t}} = \chi_{\mathrm{w}}^{\mathrm{m}} \nu_{\mathrm{w}} + (1 - \chi_{\mathrm{w}}^{\mathrm{m}}) \nu_{\mathrm{w}} \tag{42}$$

$$\mu_{\mathrm{t}} = \rho_{\beta}^{0,\mathrm{t}} \nu_{\mathrm{t}} \tag{43}$$

$$\rho_{\mathrm{t}} = \rho_{\beta}^{0} \exp\left(\beta_T \left(p_{\mathrm{m}} - p_{\beta}^{0}\right)\right) \tag{44}$$

$$\Delta p_{\mathrm{bp}} = \Delta p_{\mathrm{bp}}^{\varsigma} \left(\frac{f_{\mathrm{bp}}}{f_{\mathrm{bp},0}}\right)^2 \tag{45}$$

$$v_{\mathrm{t}} = \frac{\dot{V}_{\mathrm{t}}}{A_{\mathrm{t}}} \tag{46}$$

$$N_{\mathrm{Re,t}} = \frac{\rho_{\mathrm{t}} v_{\mathrm{t}} d_{\mathrm{t}}}{\nu_{\mathrm{t}}} \tag{47}$$

$$f_{\mathrm{D}}^{\mathrm{t}} = \frac{1}{4\left(\log_{10}\left(\frac{5.74}{N_{\mathrm{Re,t}}^{0.9}} + \frac{\varepsilon_{\mathrm{t}}/d_{\mathrm{t}}}{3.7}\right)\right)^2} \tag{48}$$

$$\Delta p_{\mathrm{f}}^{\mathrm{t}} = \ell_{\mathrm{t}} \cdot f_{\mathrm{D}}^{\mathrm{t}} \frac{\rho_{\mathrm{t}}}{2} \frac{v_{\mathrm{t}}^2}{d_{\mathrm{t}}}. \tag{49}$$

Again, we need to know the initial condition of the differential variable ($\dot{V}_{\mathrm{t}}$), the inputs ($\chi_{\mathrm{w}}^{\mathrm{m}}$, f_{bp}, p_{m}, p_{s}), and the parameters.

3.5 *Combined model*

For illustration, we use two vertical pipes, one manifold, and one horizontal transport pipe from manifold to separator. Both Modelica and Julia's ModelingToolkit have support for building classes/reusable models. Because of the similarity between the models for vertical and horizontal pipes, it would be possible to collect these in the same class/constructor and just differentiate between them with a function argument. The manifold model should be a separate class, though.

With re-usability of such classes/constructors, modeling of the combined system simply consists of (i) instantiating one model per unit (2 vertical pipes, one horizontal transport pipe, and the manifold), and (ii) connecting the various instances. Specifically, the vertical pipes should see the same manifold pressure p_{m}, the vertical transport pipe should have the same inlet pressure as the manifold pressure p_{m}, the influent volumetric flows to the manifold should be the sum of the flows from the vertical pipes and the viscosity diluting water feed $\dot{V}_{\mathrm{w}}$ now being

$$\dot{V}_{\mathrm{w}} = \frac{\sum_{i=1}^{2} \left(\chi_{\mathrm{w}}^{\mathrm{m}} - \chi_{\mathrm{w}}^{i}\right) \dot{V}_{\mathrm{v}}^{i}}{1 - \chi_{\mathrm{w}}^{\mathrm{m}}}; \tag{50}$$

the effluent volumetric flow from the manifold is still $\dot{V}_{\mathrm{t}}$.

For a proper re-usable implementation, connections should be done using *connectors* (supported by both Modelica and ModelingToolkit).

4 Simulation tools

The combined model has been solved using the free languages/tools OpenModelica (Fritzson, 2015; Fritzson et al., 2018) and ModelingToolkit (Ma et al., 2021) for Julia; the results are identical plus/minus variations due to solver accuracies. Results presented in Section 5 use the ModelingToolkit/Julia implementation due to better support in Julia for plotting and random variables.

Modelica is a mature language dating back to the 1990s; ModelingToolkit is some 2–3 years young and is still in some flux. ModelingToolkit is evolving rapidly, is more general than Modelica, and is also integrated in the larger Eco-system of Julia. Currently, ModelingToolkit does not support a graphical flow-sheeting tool, and it is unclear whether ModelingToolkit allows for as large models as OpenModelica. Both tools have extensive support for building libraries.

The default solver in OpenModelica is excellent, although here it struggled with the DAE formulation with momentum as differential variable. ModelingToolkit can use solvers from the large, high quality DifferentialEquations.jl package (Rackauckas & Nie, 2017). With ModelingToolkit, choice of solver, accuracies, etc., currently requires more thought compared to OpenModelica. The solutions from ModelingToolkit include interpolation functions, which yields smooth solutions with fewer data points.

OpenModelica normally works well when providing initial conditions for differential variables only, while with ModelingToolkit it is necessary to also specify initial values for algebraic variables.

OpenModelica's support for linearization and plotting can be accessed from Julia via the OMJulia API (B. Lie et al., 2019). ModelingToolkit is integrated in the Julia Eco-system, with support for linearization, plotting, control systems analysis, random variables, etc., and has overall more possibilities that OpenModelica if further analysis is required.

Other commonly used languages for scientific computing are MATLAB (commercial) and Python (free). Compared to both of these languages, Julia (free) has a more extensive set of differential equation solvers. Neither MATLAB nor Python offer equation based modeling languages with library/re-use support such as Modelica or ModelingToolkit; MathWorks do offer Simscape[8] (commercial) for such use, though.

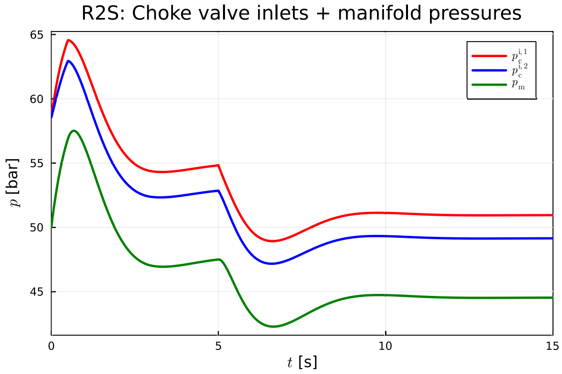

Figure 2. Pressures in front of choke valve into manifold for vertical pipes (red, blue) and manifold pressure (green).

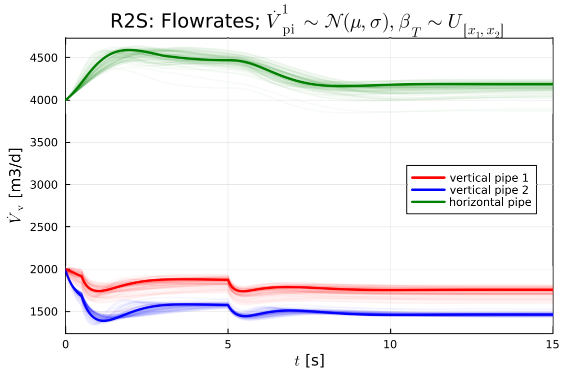

Figure 3. Vertical flow rates (red, blue) from bore-well into manifold, and horizontal flow rate (green) from manifold to separator, with uncertainty productivity capacity and isothermal compressibility.

5 Results

Parameters, initial conditions, and system inputs are given in Appendix A. For vertical pipe #2, scaling pump head h_{p}^{ς} is set to 80% of the value suggested in the appendix. Figure 2 shows the pressures in front of the choke valves for the vertical pipes, as well as the manifold pressure. The resulting time constants and overall behavior in Fig. 2 are similar to those in Sharma (2014).

Figure 3 shows vertical flow rates from reservoir to manifold in the two pipes, as well as the flow from manifold to separator (thick, solid lines), and the effect of uncertain productivity indices in Well 1, $\dot{V}_{\text{pi}}^{1} \sim \mathcal{N}\left(7\cdot 10^{-4}, 10^{-4}\right)$, and uncertain isothermal compressibility in the petroleum fluid, $\beta_T \sim U_{\left[0.3/1.5\cdot 10^{9}, 3/1.5\cdot 10^{9}\right)]}$.

ModelingToolkit has support for efficient Monte Carlos studies; this is comparatively more complicated using Modelica + OMJulia.

Both Modelica+OMJulia and ModelingToolkit have similar possibilities to linearize the models, and Con-

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[8] https://se.mathworks.com/products/simscape.html

trolSystems.jl for Julia has similar capabilities as MATLAB's Control Toolbox for plotting and analysis/design.

6 Conclusions

This paper presents a simple model of production of liquid petroleum (oil+water) from reservoir to separator. The model is essentially a reworking of the model in Sharma & Glemmestand (2014). Modifications include: (i) a stricter utilization of the constant density assumption in pipes[9] leading to a more realistic behavior at choke valves, (ii) rephrasing of algebraic equipment models into dimensionless form, which greatly simplifies unit conversion, (iii) streamlining of the model presentation to ease the implementation of the model; in the original formulation, some information is missing, and information is spread through a long paper, (iv) scaling down the model from 4 vertical pipes/2 horizontal pipes to 2 vertical pipes/1 horizontal pipe.

The model is implemented in OpenModelica and in Julia with ModelingToolkit. These tools have similar capabilities, although Modelica is more mature, has perhaps better default solver, and can handle larger systems at the moment. However, ModelingToolkit is embedded in the larger Eco-system of Julia, with superior capabilities for plotting, uncertainty analysis[10], simpler linearization, control analysis and design, etc. Combining OpenModelica with OMJulia, some of the features of the Julia Eco-system can be utilized (plotting, linearization, etc.). However, with ModelingToolkit, other tools in Julia have access to the symbolic form of the model, and can symbolically compute Jacobians, etc. Both of the free tools OpenModelica and ModelingToolkit are equation based modeling languages with solid support for model libraries and re-use of code.

The presented model was developed for short-term industrial oil production control design, Sharma (2014). More comprehensive models typically include a reservoir model (time constant: months+) suitable for long-term simulation studies (K.-A. Lie, 2019), with a steady state network solver for the transport from reservoir to separator (time constant: seconds+), thereby avoiding stiffness issues. These steady state models are not really suitable for control design for daily operation, while the model presented here has been used to assess industrial control policies.

A number of possible extensions for the system include (a) more realistic properties (density, viscosity), (b) allowing for distributed density along pipes[11], (c) adding a more realistic system for water dilution in the manifold, (d) inclusion of valves in manifold–separator pipes, (e) integration with reservoir models, (f) use for control design, (g) use for optimization, etc. Such extensions will give more insight into the industrial usefulness of the model.

Table 1. Parameters: petroleum liquid.

Parameter
$\beta_T = \frac{1}{1.5\cdot 10^9} \approx 6.67\cdot 10^{-10}\,\mathrm{Pa}^{-1}$
$p_0 = 1\,\mathrm{bar}$
$\rho_\mathrm{o} = 900\,\mathrm{kg/m^3}$
$\rho_\mathrm{w} = 1000\,\mathrm{kg/m^3}$
$\chi_\mathrm{w} = 0.35$
$\rho_0 = \chi_\mathrm{w}\rho_\mathrm{w} + (1-\chi_\mathrm{w})\,\rho_\mathrm{o}$
$\chi_\mathrm{w}^\mathrm{m} = 0.5$
$\rho_0^\mathrm{m} = \chi_\mathrm{w}^\mathrm{m}\rho_\mathrm{w} + (1-\chi_\mathrm{w}^\mathrm{m})\,\rho_\mathrm{o}$
$\nu_\mathrm{o} = 100\,\mathrm{cSt} = 100\cdot 10^{-6}\,\mathrm{m^2/s}$
$\nu_\mathrm{w} = 1\,\mathrm{cSt} = 10^{-6}\,\mathrm{m^2/s}$

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A Parameters and Operating Conditions

Parameters for petroleum fluid, nominal vertical pipes, and nominal manifold+horizontal pipe are given in Tables 1–3. Initial states are given in Table 4, while input functions are given in Table 5.

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[9] The original model includes differential equation for the pipe mass balance, although the mass is assumed constant.

[10] Modelica lacks proper support for random numbers.

[11] ModelingToolkit for Julia has support for automatic discretization of PDEs in the works.

Table 2. Parameters: vertical pipe.

Parameter
$\ell^{-} = 100\,\mathrm{m}$
$\ell^{+} = 2000\,\mathrm{m}$
$d = 0.1569\,\mathrm{m}$
$\varepsilon = 0.0018\,\mathrm{inch} = 45.7\,\mu\mathrm{m} = 45.7 \cdot 10^{-6}\,\mathrm{m}$
$h_{\mathrm{p}}^{\varsigma} = 1210.6\,\mathrm{m}$
$f_{\mathrm{p},0} = 60\,\mathrm{Hz}$
$\dot{V}^{\varsigma} = 1\,\mathrm{m}^3/\mathrm{s}$
$a_1 = -37.57$
$a_2 = 2.864 \cdot 10^3$
$a_3 = -8.668 \cdot 10^4$
$\dot{m}_{\mathrm{v}}^{\mathrm{c}} = 25.9 \cdot 10^3\,\mathrm{kg}/\mathrm{h}$
$f(u_{\mathrm{v}}) = \begin{cases} 0, & u_{\mathrm{v}} \leq 0.05 \\ \frac{11.1u_{\mathrm{v}}-0.556}{30}, & 0.05 < u_{\mathrm{v}} \leq 0.5 \\ \frac{50u_{\mathrm{v}}-20}{30}, & 0.5 < u_{\mathrm{v}} \leq 1 \end{cases}$
$p^{\varsigma} = 1\,\mathrm{bar}$
$\rho^{\varsigma} = 1000\,\mathrm{kg}/\mathrm{m}^3$
$\dot{V}_{\mathrm{pi}} = 7 \cdot 10^{-4}\,\mathrm{m}^3/\mathrm{s}$

Table 3. Parameters: manifold+horizontal pipe.

Parameter
$\ell_{\mathrm{m}} = 500\,\mathrm{m}$
$d_{\mathrm{m}} = 0.1569\,\mathrm{m}$
$\ell_{\mathrm{t}} = 4000\,\mathrm{m}$
$d_{\mathrm{t}} = 0.1569\,\mathrm{m}$
$\varepsilon = 0.0018\,\mathrm{inch} = 45.7\,\mu\mathrm{m} = 45.7 \cdot 10^{-6}\,\mathrm{m}$
$\Delta p_{\mathrm{bp}}^{\varsigma} = 10\,\mathrm{bar}$
$f_{\mathrm{bp},0} = 60\,\mathrm{Hz}$

Table 4. Nominal initial states.

Variable
$\dot{V}_{\mathrm{v}}(t=0) = 2000\,\mathrm{m}^3/\mathrm{d} \approx 0.02315\,\mathrm{m}^3/\mathrm{s}$
$p_{\mathrm{m}}(t=0) = 50\,\mathrm{bar} = 50 \cdot 10^5\,\mathrm{Pa}$
$\dot{V}_{\mathrm{t}}(t=0) = 2000\,\mathrm{m}^3/\mathrm{d} \approx 0.02315\,\mathrm{m}^3/\mathrm{s}$

Table 5. Nominal inputs.

Variable
$p_{\mathrm{f}}(t) = \begin{cases} 220\,\mathrm{bar}, & t < 0.5\,\mathrm{s} \\ 0.95 \cdot 220\,\mathrm{bar}, & t \geq 0.5\,\mathrm{s} \end{cases}$
$p_{\mathrm{s}}(t) = \begin{cases} 30\,\mathrm{bar}, & t < 3\,\mathrm{s} \\ 0.97 \cdot 30\,\mathrm{bar}, & t \geq 3\,\mathrm{s} \end{cases}$
$f_{\mathrm{p}}(t) = \begin{cases} 60\,\mathrm{Hz}, & t < 5\,\mathrm{s} \\ 0.95 \cdot 60\,\mathrm{Hz}, & t \geq 5\,\mathrm{s} \end{cases}$
$u_{\mathrm{v}}(t) = 1.0$
$f_{\mathrm{bp}} = 60\,\mathrm{Hz}$

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